The Dissertation Committee for Shaojin Li
certifies that this is the approved version of the following dissertation:

ESSAYS ON INVESTMENT AND ADVERSE SELECTION

Committee:

Russell W. Cooper, Supervisor

Philip D. Corbae, Supervisor

John C. Haltiwanger

Eugenio J. Miravete

Thomas E. Wiseman

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Shaojin Li, M.S.

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Supervisors: Russell W. Cooper
Philip D. Corbae

Relative used capital price, the measure of irreversibility, is fixed in almost all the investment literature. This dissertation introduces investment models with state-dependent irreversibility and tests whether these models outperform fixed irreversibility cases, at both the macro and micro levels. Since there is currently no historical data available on the issue of used capital prices, the first chapter uses an indirect inference procedure to estimate the cyclical property of irreversibility at the micro-level. In the second chapter, I propose a dynamic investment model with endogenous irreversibility arising from the lemons problem in the used capital market and examine the cyclical implication of irreversibility. Data evidence shows that capital reallocation, or used capital expenditure, is pro-cyclical. In a general equilibrium framework, the third chapter reveals that the investment model with state-dependent irreversibility explains this phenomenon while the model with fixed irreversibility does not.

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Chapter 1

Fixed or State-dependent Irreversibility

1.1 Introduction

Irreversibility is the property of capital that plants have to sell their capital with a price lower than their purchasing price. The degree of irreversibility is assumed to be fixed across time and states in the previous investment literature. This paper employs an indirect inference method to test whether the level of irreversibility varies with aggregate states. The empirical results support pro-cyclical irreversibility. That is, the relative used capital price, as the measure of irreversibility, is pro-cyclical.

The fixed irreversibility assumption is inappropriate from the theoretical perspective. The underlying explanation for the fixed irreversibility is capital specificity. However, irreversible investment may result from lemons problem in the used capital market. As pointed by Dixit and Pincyck in "Investment Under Uncertainty",

"*Even investments that are not firm or industry specific are often partly irreversible because buyers in markets for used machines, unable to evaluate the quality of an item, will offer a price that corresponds to the average quality in the market. Sellers, who know the quality of the item they are selling, will be reluc-*

tant to sell an above-average item. This will lower the market average qulity, and therefore the market price."

The irreversibility arising from lemons problem varies with aggregate states. Investment decisions depend on the degree of adverse selection in the used capital market while the magnitude of adverse selection relies on plants' investment behaviors. In the presence of aggregate shocks, plants' investment decision varies with aggregate states, so does the degree of adverse selection. Therefore, the level of irreversibility varies with the aggregate states.

Although lemons problem allows the irreversibility to vary with aggregate states, previous studies still assume the irreversibility is fixed over time and across states without testing the hypothesis. Due to the difficulty of collecting data, to estimate the degree of irreversibility directly is rare and limited to some industry. Using equipment-level data of aerospace industry, Ramey and Sharpiro (2001) report the level of irreversibility is about .75 during the nineties. Other studies (Cooper and Haltiwanger(2006), Bloom (2008), etc) consider irreversibility as one of capital adjustment costs. They estimate the fixed level of irreversibility along with parameters of other capital adjustment costs indirectly, using simulated method of moments. That is, they choose the parameters which best fit micro-level investment moments.

This paper also employs indirect inference procedure to estimate the cyclical component of irreversibility along with other forms of capital adjustment costs. The present paper differs from previous studies in two aspects. First, this paper assumes that the irreversibility, or the relative used capital price, is a function of aggregate shocks. By choosing a functional form which nests the fixed irreversibility case, the proposed investment model can test whether the irreversibility varies with aggregate states. Second, the present paper introduces an investment moment with cyclical properties to pin down the state-dependent irreversibility. Based on the observations in Longitudinal Research Database (LRD), the fraction of inactive plants is negatively correlated with aggregate output.

The empirical results indicate that the irreversibility has a cyclical component and the relative used capital price, as a measure of irreversibility, is procyclical. The rest of the paper organizes as follows: Section 1.2 describes the state-contingent investment model; Section 1.3 outlines the estimation procedure and results; Section 1.4 concludes.

1.2 The Model

To estimate the cyclical component of irreversibility, this section introduces an investment model with state-contingent irreversibility. Unlike the previous models (Cooper and Haltiwanger (2006) and Bloom (2008)), the difference between the price that the plant can buy capital goods and the price that the plant can sell installed capital varies with aggregate states. The aggregate states are captured by aggregate shocks A_t, which follow an AR(1) process in logs. The value with prime stand for that next period.

$$logA' = \rho_A logA + \upsilon'_A \text{ with } \upsilon_A \sim N(0, \sigma_A^2)$$

The function form is chosen so that the state-dependent irreversibility model nests the model with fixed irreversibility.

$$p_{s,t}(A) = \frac{1}{1 + exp(\alpha_1 + \alpha_2 A)}$$

$p_{s,t}$ is the selling price of capital at period t. A_t is the aggregate shock at period t. $p_{s,t} \in [0, 1]$. When $\alpha_2 = 0$, $p_{s,t}$ is a constant, which is the case with fixed irreversibility. In addition to aggregate technology shocks, the plants face idiosyncratic shocks a_t, which also follows an AR(1) process in logs. The aggregate and idiosyncratic shocks are independent and multiplicative.

$$loga' = \rho_z loga + \upsilon'_z \text{ with } \upsilon_z \sim N(0, \sigma_z^2)$$

In addition to irreversibility, other kinds of capital adjustment costs affect plant investment behaviors. In this paper, I also estimate convex adjustment costs and non-convex adjustment costs. The convex adjustment cost has a quadratic form. The non-convex adjustment cost is proportional to the profit of the plant. It stands for the destruction cost when investing or disinvesting capital. The plant is characterized by the idiosyncratic shock a, the aggregate shock A and its predetermined capital stock. The one-period profit function of a plant with a triple (a, A, k) is defined as follows:

$$\pi(a, A, k) = aAk^{\alpha}$$

The plant's investment problem is described in equation (1.1), (1.2) and (1.3).

$$V(a, A, k) = max\{V^a(a, A, k), V^i(a, A, k)\} \ \forall(a, A, k) \tag{1.1}$$

$$
\begin{aligned}
V^a(a,A,k) = max_{k'}\lambda\pi(a,A,k) - \frac{\nu}{2}((k'-(1-\delta)k)/k)^2k \\
- \mathbf{1}_{k'>(1-\delta)k}(k'-(1-\delta)k) - \mathbf{1}_{k'\leq(1-\delta)k}p_s(A)(k'-(1-\delta)k) \quad (1.2) \\
+ \beta E_{a',A'|a,A}V(a',A',k') \\
V^i(a,A,k) = \pi(a,A,k) + \beta E_{a',A'|a,A}V(a',A',k(1-\delta)) \quad (1.3)
\end{aligned}
$$

The buying price of capital is normalized to 1. The subscript "a" stands for the act of investing, "i" for inaction. k is the predetermined capital stock at current period. β is the annual discount rate . The annual depreciation rate is δ .

1.3 Estimating the Model

The main purpose of the paper is to estimate the cyclical component of the degree of irreversibility. Besides the related parameters, the parameter vector θ of in interest characterizes the the profit function, stochastic processes, convex adjustment cost and non-convex adjustment cost. Because of no access to micro-level data, this paper employs an indirect inference procedure to estimate the parameters which minimize the distance between key moments and estimates from the simulated data and the actual data, reported in Cooper and Haltiwanger (2006). The discount rate and depreciation rate are predetermined.

1.3.1 Two-Step Indirect Inference Procedure

The indirect inference procedure consists of two steps[1]. The parameters of interest are divided into two groups. The first group includes the curvature of the

[1] An alternative method is to estimate the parameter vector simultaneously. As the indirect inference procedure includes a two-step GMM estimation, the computation load is extremely high.

profit function and the parameters of the shock processes, $\theta_1 = (\alpha, \rho_A, v_A, \rho_z, v_z)$. The second group includes the parameters of adjustment costs, $\theta_2 = (\lambda, \alpha_1, \alpha_2, \nu)$. The first step of the estimation procedure is to estimate θ_2 using simulated methods of moments, given an initial guess of θ_1, and generate a panel of simulated data. The second step is to estimate θ_1 using the simulated data. Then compare the distance with the estimates from actual data. If it is very close, then stop the iteration. Otherwise update the parametrization of θ_1 and start the loop again.

At the step of SMM, a set of data moments μ^d is chosen for the model to match. Given an arbitrary parametrization of (θ_1, θ_2), the optimal investment policy is derived from solving the plant investment problem. This optimal policy is used to simulate a panel with N plants and T periods. This procedure is replicated for S times. The simulated moments μ^s are the average of these replications. In the empirical exercise, the time period T is 15 years as the actual data is 15 year long. Because of the computation load, The simulated number of plants is chose to be 500, which is less than that in the real data. When increasing the number of simulated plants, the empirical result does not change a lot.

The estimated $\hat{\theta_1}$ is then the parameter vector which minimizes the following objective function given θ_2.

$$\hat{\theta_1} = argmin_{\theta_1 \in \Theta_1} (\mu^d - \mu^s(\theta_1))' W (\mu^d - \mu^s(\theta_1)) \tag{1.4}$$

W in equation (1.4) is the weighting matrix. In the empirical exercise, I use an identity matrix[2]. Due to the potential for discontinuities of the objective function in

[2]In this paper, I do not use the optimal weight matrix because an estimation of variance-

equation (1.4), a simulated annealing algorithm is used to find the global minimization of the objective function.

The estimation of the parameter of non-convex adjustment cost is related to the estimation of parameters of shocks. When plants adjust their capital stock, they encounter some destruction costs so that the profit with capital adjustment is proportional to the profit without adjustment. $\lambda \in [0, 1]$ is the ratio. As econometricians only observe the realized profit, it is hard to separate λ from the profit function. Assuming $\lambda = 1$, Cooper and Haltiwanger (2006) estimate the parameters of shocks and profit function based on the following quasi-difference function. In their estimates, the curvature of the profit function $\alpha = 0.592$. $\rho_A = 0.76$, $\sigma_A = 0.05$, $\rho_z = 0.885$, $\sigma_z = 0.30$.

$$log(\pi_{i,t}) = \rho_z log(\pi_{i,t-1}) + \alpha log(k_{i,t}) - \rho_z \alpha log(k_{i,t-1}) + log(A_t) - \rho_z log(A_{t-1}) + v_{z,i}^t \tag{1.5}$$

At the second step of estimation, I re-estimate θ_2 using the simulated data assuming $\lambda = 1$. In equation (1.5), the aggregate shocks are unobservable. Thus, a complete set of year dummies is used as their proxies in estimation. As described in Cooper and Haltiwanger (2006), the above equation is estimated via General Methods of Moments (GMM). Lagged and twice lagged capital stock and twice lagged profits are used as instruments. The weighting matrix is optimal.

covariance matrix of actual moments conditions is necessary to calculate the optimal weighting matrix, as shown in page 88 in Adda and Cooper (2003) and page 31 in Gourieroux and Monfort(1996). However, I do not have the access to individual observations. An alternative method is to assume that the data moments have the same covariance matrix as their simulated counterparts. In this case, the computation load is too high when employing a two-step iteration to calculate the optimal weighting matrix.

1.3.2 Identification

As shown in Gourieroux and Monfort(1996), the estimators of indirect inference is consistent. But the moment conditions have to be informative about the parameters of interest. As the focus of the paper is the cyclical component of irreversibility, the choice of moment conditions have to reflect the impact of state-contingent properties. In the analysis of Caballero (1999), he states that the impact of irreversible investment is on the inactive behaviors. Thus, I choose the correlation of fraction of inactive plants and aggregate states as the moment condition.

The number of inactive plants varies over the cycles. The row "inacta" in Table A.1 lists the fraction of inactive plants from 1974 to 1988. Inactive plants are those whose invest or disinvest rates are less than 1%. These numbers are provided by John Haltiwanger from the calculation of Cooper and Haltiwanger (2006). In order to explore the co-movement with output, I use linear trend to separate cyclical components of inaction rates and real GDP(Gross Domestic Production). Figure A.6 plots their percentage deviations. From the figure, the inaction rate is high when output is low. That is, the inaction rate is negatively correlated with output. The correlation is -0.54598.

As we can not calculate the counterpart of total output from the partial equilibrium investment model, I use the correlation of average profit shocks with inaction rate as the proxy of the correlation between total output and inaction rates. There are two reasons. First, the average profit shock is procyclical. The correlation between the average profit shock and total output from 1974 to 1988 is 0.84245. Second, the average profit shock across plants is approximately the aggregate profit shock. As shown in Figure A.6, the average profit shock is high when the inaction rate is low. The correlation is -0.60749.

Other kinds of adjustment costs also affect the inaction rate over cycles. Therefore, I estimate the convex and non-convex adjustment costs along with the degree of irreversibility. The choice of other moment conditions follows Cooper and Haltiwanger (2006), which use the correlation of investment rate and profit shock, the autocorrelation of investment rate, the positive and negative spike rate[3]. Those moments are based on observations in Longitudinal Research Database(LRD) which consists of about 7000 manufacturing plants in operation between 1972 and 1988[4].

1.3.3 Estimates of Irreversibility

This section presents our estimation results. Table A.2 summarizes main results of this paper[5]. The empirical results provide substantial, statistically significant evidence for state-contingent irreversibility. Estimated values of α_1 and α_2 are 32.06 and -36.27 respectively in Table A.2. They are both significantly different from zero. The rest of results support relative modest convex and nonconvex adjustment costs. Estimate of λ is 0.8370 which is close to 0.796 in Cooper and Haltiwanger (2006). Our estimate of ν is relatively low, 0.031, which indicates convex adjustment cost plays a small role in our model.

[3]The positive (negative) spike rate is the fraction of investment rate which is larger (less) than $20\%(-20\%)$.

[4]After computing the investment rate and the autocorrelation of investment rate, the time period is 15 years.

[5]For this result, the curvature of profit function is 0.588. The serial correlation of aggregate and idiosyncratic shocks are 0.692 and 0.90 respectively. The conditional volatilities of the shocks are 0.0545 and 0.28 respectively.

The used capital price as the measure of the degree of irreversibility is pro-cyclical. When the whole economy faces a positive technology shock, the used capital price increases. That is, the capital adjustment cost decreases when selling capital. On the contrary, the convex and non-convex adjustment costs increase in the boom.

1.4 Conclusion

This paper estimates an investment model with state-contingent irreversibility in a partial equilibrium framework. Estimation results support pro-cyclical price premium between the selling price and buying prices of capital. As we know, the price premium is determined by the interaction between new and used capital markets and it is affected by the information structure in used capital market. To deeply understand the nature of capital adjustment, an explicit model in a general equilibrium framework will be helpful to analyze different forces in these markets and evaluate the impact of aggregate uncertainty on the price premium quantitatively.

Chapter 2

State-dependent Irreversibility and Lemons Market

2.1 Introduction

Lemons problem in the used capital market is one of the main explanations for partial irreversible investment. But previous literature does not study the determination of the degree of irreversibility arising from lemons problem. As the magnitude of irreversibility depends on plant investment decisions, this kind of investment models predicts state-contingent irreversibility in the presence of aggregate shocks. What is the cyclical property of irreversibility arising from lemons problem? Are the model predictions consistent with data ? This chapter develops an investment model with adverse selection and answers these questions.

Although previous studies assume that the degree of irreversibility is fixed, there are some evidences supporting the cyclical irreversibility. First, as shown in the first chapter, the relative used capital price, as the measure of the degree of irreversibility, is procyclical. When the economy faces a common positive shock, the used capital price is relatively higher. Second, according to the Annual Capital Expenditures Survey, the fraction of used capital expenditure over total capital expenditure is countercyclical. That is, when the economy faces a common positive shock, the demand of the used capital is lower relative to the demand of the new capital. In this chapter, I propose a framework to study the investment behaviors in the presence of adverse selection in the used capital market and characterize the optimal investment policy analytically. In a calibrated economy, the implied cyclical property of used capital price is consistent with the empirical results in the first chapter. The fraction of used capital expenditure is also countercyclical.

This paper models the impact of adverse selection on investment behaviors explicitly. In the model economy, capital has two types: good quality or bad quality, which are unobservable to buyers. Plants can invest in new and used capital market, but can only disinvest in the used capital market. The quality of new capital is exogenous while that of used capital is endogenous. No-arbitrage condition makes plants indifferent between two markets. Since the quality of used capital is higher than that of bad capital, owners of lemons always sell out bad capital. Since the quality of used capital is lower than that of good capital, plants never buy used capital and sell good capital simultaneously. Besides selling lemons, plants have three kinds of actions: selling good capital, inaction (no selling good capital and no purchasing capital) and purchasing capital.

The optimal investment decisions follow a two-side sS rule. When the idiosyncratic shock of an plant is less than the lower threshold, the plant sells good capital. When the idiosyncratic shock is above the upper threshold, the plant purchases capital. If the idiosyncratic shock is between the two thresholds, the plant only sells bad capital. The two thresholds are contingent to the market conditions: the used capital price and plants' beliefs about the average quality in the used capital market.

In a pooled used capital market, the quantities of capital sold equal the quantities of capital purchased, which derives the equilibrium used capital price. The qualities of the capital sold determines average quality in the market. In equilibrium, plants' beliefs are consistent with the realized average quality in the market. That's how irreversibility, as measured by used capital price, interacts with plants' investment decisions in the presence of asymmetric information in the used capital market.

Existing plants sell used capital for two reasons: hold lemons or face low idiosyncratic shocks. This feature of the model avoids the breakdown of used capital market in Akolof(1970). The used capital price is related to both quantities and qualities of capital sold in the market. As the quality and price of new capital are fixed[1], the used capital price increases with average quality in the market, or everyone go to the new capital market.

The interaction between two assets and combination between sS adjustment and adverse selection are closely related to the works in House and Leahy(2004) and Eisfeldt(2004). House and Leahy(2004) studies the impact of adverse selection in a used durable market. They find sS bands shrink when increasing the variance of taste shock. Eisfeldt(2004) uses adverse selection to explain the illiquid long term risky asset in the equity market. The comparative statics show that liquidity increases with productivity level.

The model with endogenous irreversibility predicts state-dependent irre-

[1]The new capital price is numeraire in the model.

versibility in a world with aggregate uncertainty. Different uncertainty level affects the thresholds of sS rule. Plants' investment decisions are also contingent to aggregate uncertainty. The corresponding irreversibility hence has cyclical component. The capital adjustment cost from irreversibility is also endogenous and state-dependent. Eisfeldt and Rampini (2005) find only countercyclical capital reallocation costs can explain the coexistence of procyclical capital reallocation and countercyclical benefit in the firm-level. As all the physical adjustment costs are procyclical, they infer that the countercyclical costs come from "informational and contractual" frictions. The cyclical implication of the present model is consistent with their findings.

The irreversibility arising from lemons problem is countercyclical. The positive technology shock increases the demand for used capital and decreases the incentives of incumbents to sell good capital. Used capital price goes up while the fraction of good capital in the used market tends to decline. The no-arbitrage condition between new and used capital markets requires the used capital price increases with rising quality. In the calibrated model, plants invest more on new capital which has higher quality at the good state. Thus average quality of exiting plants is higher if aggregate uncertainty is persistent. The sales from exiting plants compensate the declining quality in the market. Used capital price and fraction of good capital are both high in the boom. Thus investment is more reversible in the boom and capital reallocation is procyclical.

The remaining parts of the paper are organized as follows. Section 2.2 describes the model. Section 2.3 characterizes the properties of decision rules and equilibria. Section 2.4 explores the cyclical properties of irreversibility. Section 2.5 concludes.

2.2 The Model

The model embeds asymmetric information in the used capital market into a two-period investment problem. Plants can invest in new and used capital markets, but only disinvest in the used capital market. The private information about the quality of capital causes the relative used capital price less than one. The capital adjustment cost occurs endogenously. The interaction between two markets and the adverse selection determine the equilibrium used capital price, i.e. the degree of irreversibility. The higher the price is, the less irreversible the investment is.

2.2.1 Production and Quality of Capital

Capital is the only input of production. The quality of capital describes the extent of utilization of capital in production. The quality of good capital is 1, and that of bad capital is $\phi \in (0, 1)$. Plants produce the single good with the Cobb-Douglas production function ($\alpha \in (0, 1)$):

$$y = aA\tilde{k}^{\alpha}; \;\; \tilde{k} = k_g + \phi k_b$$

where k_g and k_b are good and bad capital stocks, $\tilde{k}$ is the efficiency units of total capital stock, A is the level of aggregate uncertainty and a is the plant-specific productivity. The aggregate uncertainty follows a two-state Markov chain with a stationary transition matrix. π_{ij} is the probability to have A_j next period given A_i

this period, $i, j \in \{h, l\}$. Aggregate uncertainty and plant-specific productivity are multiplicative and independent.

2.2.2 Information Structure and Used Capital Market

Plants have private information about the qualities of their capital. Buyers purchase used capital as a price taker. The key assumption is the anonymity of the used capital market. Or the actions of sellers would potentially reveal the quality of capital, for example, the quantity of capital sold, the turnover rate of capital even the observable plant-specific productivity. The used capital market acts like a middleman who trades with plants at an equilibrium price.

In a pooled used capital market, buyers purchase a bundle of good and bad capital. Denote Q by the fraction of good capital in the market. The average quality here is $Q+\phi(1-Q)$. To simplify the model, I assume that the quality of used capital equals the average quality in the market. In the new capital market, the fraction of good capital is q_0, which is exogenous. The quality of new capital equals the average quality.

$$\bar{q} = q_0 + \phi(1 - q_0)$$

2.2.3 The Investment Problem

Plants live two periods. The economy has a continuum of plants each period, half of which are young, the rest are old. At the end of each period after production, young plants make investment decisions, and old plants sell all of their capital and exit. At the beginning of next period, young plants become old. Same fraction of newborns enter the world with new capital stock k_0. Every entrant picks a productivity draw from the distribution function $F(\cdot)$, which has a positive support. At the initial period, the aggregate good and bad capital stocks from old plants are $\bar{K}_g^0$ and $\bar{K}_b^0$.

Young plants believe that the fraction of good capital in the used capital market is λ. The quality of used capital in their minds is:

$$\bar{\lambda} = \lambda + \phi(1 - \lambda)$$

They can buy new capital or used capital. They can also sell good capital or bad capital. Denote new capital expenditure by i_n; used capital expenditure by i_u; disinvestment of good capital i_g and disinvestment of bad capital i_b. The efficiency units of capital stock at the beginning of next period are:

$$\tilde{k}' = i_n\bar{q} + i_u\bar{\lambda} + k_0(1 - \delta)\bar{q} - i_g - i_b\phi$$

An individual young plant's problem at period t is described as follows:

$$max_{\{i_{n,t}\geq 0, i_{u,t}\geq 0, i_{g,t}\in[0,k_0(1-\delta)q_0], i_{b,t}\in[0,k_0(1-\delta)(1-q_0)]\}}\Pi_{1,t}+\beta E\{\Pi_{2,t+1}|A_t\}\ s.t. \quad (2.1)$$

$$\Pi_{1,t} = aA_t(k_0\bar{q})^\alpha - i_{n,t} - p_{u,t}(i_{u,t} - i_{g,t} - i_{b,t}) \quad (2.2)$$

$$\Pi_{2,t+1} = aA_{t+1}\tilde{k}_t'^\alpha + p_{u,t+1}(1 - \delta)(i_{u,t} + i_{n,t} + k_0(1 - \delta) - i_{g,t} - i_{b,t}) \quad (2.3)$$

The new capital price is numeraire. $\Pi_{1,t}$ is the profit of the plant at period t. The subscript "1" implies that the plant is young. $\Pi_{2,t+1}$ is the profit of the plants at period $t + 1$. The subscript "2" indicates that the plant is old. The subscript "t" stands for the time period t. β is the discount factor. $p_{u,t}$ is the used capital price at period t. δ is physical depreciation rate.

2.2.4 Competitive Equilibrium

The used capital price and consistent belief are equilibrium outcomes each period. In the used capital market, old plants always sell capital. Denote $\bar{K}_g^t$ and $\bar{K}_b^t$ by the aggregate good and bad capital stocks of old plants at the beginning of period t ($t > 1$),

$$\bar{K}_g^t = \int_0^\infty (i_{u,t-1}\lambda_{t-1} + i_{n,t-1}q_0 + k_0 q_0(1-\delta) - i_{g,t-1})dF$$

$$\bar{K}_b^t = \int_0^\infty (i_{u,t-1}(1-\lambda_{t-1}) + i_{n,t-1}(1-q_0) + k_0(1-q_0)(1-\delta) - i_{b,t-1})dF$$

Young plants are the potential buyers and sellers. The used capital market clearing condition at the end of period t is as follows:

$$\int_0^\infty i_{u,t}dF = \int_0^\infty i_{g,t}dF + \int_0^\infty i_{b,t}dF + (1-\delta)(\bar{K}_g^t + \bar{K}_b^t) \tag{2.4}$$

Now the fraction of good capital in the used capital market equals plants' belief.

$$\lambda_t \int i_{u,t}dF = \int i_{g,t}dF + (1-\delta)\bar{K}_g^t \tag{2.5}$$

Definition 1. *The competitive equilibria of the economy* $\{\beta, \delta, \alpha, q_0, \phi, F(\cdot), \Pi, A_h, A_l, k_0, A_0, \bar{K}_g^0, \bar{K}_b^0\}$ *are these sequences:*

$$\{p_{u,t}, \lambda_t, \{i_{n,t}(a)\}, \{i_{u,t}(a)\}, \{i_{g,t}(a)\}, \{i_{b,t}(a)\}\}_{t=0}^\infty$$

that solve plants' problems and clear the used capital market and satisfy the consistency belief condition.

- *Given* $\{p_{u,t}, \lambda_t\}$, $\{i_{u,t}(a), i_{n,t}(a), i_{g,t}(a), i_{b,t}(a)\}$ *solve (2.1)-(2.3).*

- *Given plants' decision rules, the pair $(p_{u,t}, \lambda_t)$ satisfies (2.4)-(2.5) each period.*

The supply from old plants is inelastic in the used capital market, thus the equilibrium belief is well-defined. The demand for used capital must be positive while that for new capital could be zero in equilibrium. This paper focuses on the case with positive aggregate new investment as in the reality. All the stationary equilibria have positive aggregate new investment in the paper.

2.3 Stationary Equilibria

2.3.1 The Equilibrium Belief and Price

If the fraction of good capital in the total capital stock of old plants is less than q_0 at the end of initial period. The fraction later on is bounded below q_0. The following assumption provides a sufficient condition such that the fraction of good capital in the used capital market is bounded below q_0 each period.

Assumption 1. $\frac{\bar{K}_g^0}{\bar{K}_g^0+\bar{K}_b^0} \leq q_0$.

Lemma 1. $\frac{\bar{K}_g^t}{\bar{K}_g^t+\bar{K}_b^t} \leq q_0, for \;\; all \;\; t \geq 1.$

The equilibrium belief is positive. As aggregate new investment is non-negative, the aggregate good capital stocks for each generation must be positive. Since old plants have to sell all of their capital. Some good capital is always in the used capital market. In equilibrium, the belief is consistent with the market realization. Thus the equilibrium belief is always positive. On the other side, the equilibrium belief can't exceed some upper bounds. Every plant sells its lemons since the quality of used capital is higher than that of bad capital. The fraction of good capital is below the fraction of good capital in the new capital market. The upper bound for equilibrium belief is q_0.

Proposition 2.3.1. *The equilibrium belief* $\lambda_t \in (0, q_0)$.

New and used capital are perfect substitutes in production. The ratio of marginal products equals the ratio of average qualities in two markets. In a frictionless economy, I can construct a redundant good which has the same fraction of good capital as the used capital. The price ratio between the constructed good and new capital equals the ratio of qualities. In the presence of asymmetric information, one unit of new capital has to be sold at $p_{u,t}$ no matter how high the quality is. To give plants' incentives to buy new capital, the price ratio has to be higher than their quality ratio.

Proposition 2.3.2 (no-arbitrage condition)**.** $p_{u,t} = \frac{\bar{\lambda}_t}{\bar{q}} + \frac{\bar{q}-\bar{\lambda}_t}{\bar{q}} E\beta p_{u,t+1}(1-\delta)$

From proposition 2.3.2, the lower bound of used capital price is $\frac{\bar{\lambda}_t}{\bar{q}}$. The new capital price is 1. Both new and used capital have resale value $p_{u,t+1}$ per unit next period. Since the average quality of new capital is higher than that of used capital. The used capital price should be less than new capital price. The following proposition summarizes the results.

Proposition 2.3.3. *The equilibrium used capital price* $p_{u,t} \in (\frac{\bar{\lambda}_t}{\bar{q}}, 1)$.

2.3.2 Optimal Investment Policy

Proposition 2.3.1 implies that the quality of used capital is higher than that of bad capital. Plants always sell out their lemons. The quality of good capital is higher than that of used capital, so plants do not sell good capital and buy used capital at the same time. The no-arbitrage condition indicates that plants are indifferent between new and used capital investment. Therefore the solutions to the plants' problems are not unique. But the target efficiency units of capital are unique.

$$K_1^t(a) = \left(\frac{p_{u,t} - E\beta p_{u,t+1}(1-\delta)}{\bar{\lambda}_t a\alpha E\beta A_{t+1}}\right)^{\frac{1}{\alpha-1}} \; ; \; K_2^t(a) = \left(\frac{p_{u,t} - E\beta p_{u,t+1}(1-\delta)}{a\alpha E\beta A_{t+1}}\right)^{\frac{1}{\alpha-1}}$$

$K_1^t(a)$ is the target efficiency units of capital if the plant with productivity level a purchases capital. $K_2^t(a)$ is the target if the plant sells good capital. They come from plants' first order conditions with interior solutions. Let the good capital stock of young plant at the end of period 1, $k_0 q_0(1-\delta)$, be the base point of further investment. Denote $i_{0,t}(a)$[2] by the increment of efficiency units from the base point. The investment decisions about $i_{0,t}$ and $i_{g,t}$ are thus unique.

The decision rules act like the standard sS policy. When the productivity level is below some threshold $\underline{a}_t$, the plant sells its good capital. When the productivity level is above a certain threshold $\bar{a}_t$, the plant purchases capital. When the productivity level is between two thresholds, the plant only sells bad capital.

Proposition 2.3.4 (two side (s,S) rule)**.** *In an equilibrium with positive aggregate investment, there exist two cutoff values of idiosyncratic shocks, $\underline{a}_t < \bar{a}_t$ and the solutions of plants' problems at period t are as follows:*

[2]The idiosyncratic shock a is omitted in the following text.

1. *When* $a > \bar{a}_t$, $i_{0,t} = K_1^t(a) - k_0 q_0(1-\delta), i_{g,t} = 0, i_{b,t} = (1-\delta)k_0(1-q_0)$.

2. *When* $\underline{a}_t \leq a \leq \bar{a}_t$, $i_{0,t} = 0, i_{g,t} = 0, i_{b,t} = (1-\delta)k_0(1-q_0)$.

3. *When* $a < \underline{a}_t$, $i_{0,t} = 0, i_{g,t} = k_0(1-\delta)q_0 - K_2^t(a), i_{b,t} = (1-\delta)k_0(1-q_0)$.

Graph A.4 plots a plant's optimal decision policy in a steady state. The blue line stands for plants' initial capital stock after selling all of their bad capital, $k_0 q_0(1-\delta)$. The green line plots the target level of efficiency units when selling good capital, $K_2(a)$. The red line plots the target level of efficiency units when purchasing capital, $K_1(a)$. a_1 and a_2 are cutoff values in Proposition 2.3.4. The area between $K_1(a)$ and $K_2(a)$ describes the optimal efficiency units of capital next period. When the line $k_0 q_0(1-\delta)$ is above the area, the optimal choice is to adjust capital stock downwards. When the line is below the area, the optimal choice is to buy capital. If the line falls in the area, the plant keeps its capital stock $k_0 q_0(1-\delta)$.

The capital adjustment costs arise from adverse selection in the used capital market. Sellers of lemons get adjustment subsidy since the quality of bad capital is below average quality in the used market. Sellers of good capital and buyers of used capital both face capital adjustment costs. Sellers lose one unit of efficiency capital when selling one unit of good capital, the loss in terms of new capital is $\frac{1}{\bar{q}}$. They get $p_{u,t}$. The capital adjustment cost of good capital sellers is $\frac{1}{\bar{q}} - p_{u,t}$ which is positive. When buyers get one unit of used capital, they pay $p_{u,t}$ and receive $\bar{\lambda}_t$ units of efficiency capital. The capital adjustment costs of buyers are $p_{u,t} - \frac{\bar{\lambda}_t}{\bar{q}}$ which is also positive. Thus I can use used capital price as the measure of irreversibility. The higher the used capital price is, the less irreversible the investment is.

2.3.3 Definition of Recursive Stationary Equilibrium

The aggregate state space includes aggregate good capital stock, $\bar{K}_g$, aggregate bad capital stock, $\bar{K}_b$, from old plants and aggregate uncertainty level. In many general equilibrium models, average capital stock is not the sufficient statistics for the equilibrium price. The price depends on the cross-sectional distribution that varies with aggregate uncertainty. The individual's problem has to be solved with approximation method because the distribution is infinite dimension. In the present model, although the distribution of old plants' capital stock varies, they all have to sell their capital. In an anonymous used capital market, the average levels are enough to pin down the used capital price and fraction of good capital. Denote the aggregate state vector by $X = [\bar{K}_g, \bar{K}_b, A] \in S = \mathbb{R}_+^2 \times \{A_h, A_l\}$. The individual state variable is $a \in S_a = \mathbb{R}_+$.

As mentioned before, new and used capital are perfect substitutes. The solution to plants' problem is a convex set. But the pair $(i_{0,t}, i_{g,t})$ is unique for each plant. Now think $i_{0,t}$ as a bundle of new and used capital. Suppose used capital provides x fraction of efficiency units of $i_{0,t}$. One unit of the combination good consists of $\frac{x_t}{\lambda}$ unit of used capital and $\frac{1-x_t}{\bar{q}}$ unit of good capital. Then the efficiency unit is one per bundle. After applying the no-arbitrage condition, the price of the combination good is $\frac{1-\beta(1-\delta)Ep'_u}{\bar{q}}$. The recursive formulation of plant's problems can be simplified as follows:

$$V(a;X) = max_{\{i_0,i_g\}} aA(k_0\bar{q})^\alpha + \frac{1}{\bar{q}}(-1+\beta E p_u(X')(1-\delta))i_0 + p_u(X)i_g$$
$$+ p_u(X)k_0(1-q_0)(1-\delta) + \beta E(aA'\tilde{k'}^\alpha + p_u(X')(1-\delta)(k_0q_0(1-\delta)-i_g))$$
$$s.t.\ \tilde{k'} = i_0 + k_0q_0(1-\delta) - i_g$$

$$i_0 \geq 0; i_g \in [0, k_0(1-\delta)q_0]; \quad \bar{K}_g' = H_1(X); \quad \bar{K}_b' = H_2(X)$$

x_t doesn't appear in the plants' problem. It is constructed to clear the used capital market. The demands for used and new capital are:

$$i_{u,t} = \frac{x_t i_{0,t}}{\bar{\lambda}_t}, \quad i_{n,t} = \frac{(1-x_t)i_{0,t}}{\bar{q}}$$

x, as a function of aggregate state vector, is also part of equilibrium outcomes.

Definition 2. *The stationary recursive competitive equilibrium with aggregate positive investments is characterized as these functions: used capital price* $p_u : S \to (0,1)$*, consistent belief* $\lambda : S \to (0, q_0)$*, fraction of efficient investment on used capital* $x : S \to (0,1)$*, laws of motions of average good and bad used capital stock of old plants,* $H_1 : S \to S$ *and* $H_2 : S \to S$*, value function* $V : S_a \times S \to \mathbb{R}_+$*, decision rules* $i_0 : S_a \times S \to \mathbb{R}_+$*,* $i_g : S_a \times S \to [0, k_0 q_0(1-\delta)]$*such that given* X*:*

- (V, i_0, i_g) *satisfy the above plants' maximization problem given* (p_u, λ, H_1, H_2).
- (p_u, λ, H_1, H_2) *satisfy no-arbitrage condition.*

$$p_u(X) = \frac{\bar{\lambda}(X)}{\bar{q}} + \frac{(\bar{q} - \bar{\lambda}(X))\beta(1-\delta)E[p_u(X')|X]}{\bar{q}} \tag{2.6}$$

- *Used capital market clears.*

$$\begin{aligned} \int_0^\infty \frac{(xi_0)(a;X)}{\bar{\lambda}} dF(a) =& k_0(1-q_0)(1-\delta) + \int_0^\infty i_g(a,X)dF(a) \\ &+ (\bar{K}_g + \bar{K}_b)(1-\delta) \end{aligned} \tag{2.7}$$

- *Plants' belief* λ *is consistent with* (i_0, i_g).

$$\lambda(X) \int_0^\infty \frac{(xi_0)(a;X)}{\bar{\lambda}} dF(a) = \int_0^\infty i_g(a,X)dF(a) + \bar{K}_g(1-\delta) \tag{2.8}$$

- *Laws of motions of $\bar{K}_g$ and $\bar{K}_b$ are generated by i_0, i_g, λ and x.*

$$\begin{aligned} H_1(X) = \int_0^{\infty} [(\frac{(x\lambda)(X)}{\bar{\lambda}(X)} + \frac{(1-x(X))q_0}{\bar{q}})i_0(a;X) + k_0 q_0(1-\delta) \\ - i_g(a;X)]dF(a) \end{aligned} \tag{2.9}$$

$$H_2(X) = \int_0^{\infty} [(\frac{(x(1-\lambda))(X)}{\bar{\lambda}(X)} + \frac{(1-x(X))(1-q_0)}{\bar{q}})i_0(a;X)]dF(a) \tag{2.10}$$

The existence of equilibrium depends on the parameterizations of the model. In a case with two idiosyncratic shocks without aggregate uncertainty, Proposition B.2.1 in appendix B.2 describes the sufficient conditions for the existence of equilibrium. For more complicated case, to find the existence conditions for the fixed point with five functionals is hard. The solution strategy is to assume that the equilibrium exists, compute it and check whether the solution satisfies the equilibrium conditions.

2.3.4 Computation Algorithm

The solution of the equilibrium is a fixed point of five functionals $(p_u, \lambda, x, H_1, H_2)$ which satisfy equation (2.6),(2.7),(2.8),(2.9) and (2.10) for any X. The computation algorithm mainly bases on projection method.

Let $B(X)$ be the space of functions: $p_u : S \to (0,1)$. Define an operator $T : B(S) \to B(S)$ as

$$Tp_u(X) = \frac{\bar{\lambda}(X)}{\bar{q}} + \frac{(\bar{q} - \bar{\lambda}(X))\beta(1-\delta)E[p_u(X')|X]}{\bar{q}}; \ \ given \ \ H_1 \ \ and \ \ H_2.$$

T satisfies Blackwell's sufficient conditions thus is a contraction mapping. There exists a unique p_u given the initial guess of H_1, H_2, λ. The solution reduces to a fixed point with four functionals.

Proposition 2.3.5. *T is a contraction mapping.*

As usual, the first step is to discretize the state space. If divide the state space by many rectangulars, in some grid points, $\bar{K}_g$ is high and $\bar{K}_b$ is low, which does not satisfy Lemma 1. The resulted λ may be larger than q_0. T may not have a unique fixed point and p_u may be larger than one[3]. To avoid this case, I use $r_{kb} = \frac{\bar{K}_b}{\bar{K}_g + \bar{K}_b}$ as one dimension of aggregate state vector instead of $\bar{K}_b$. According to Lemma 1, $r_{kb} \in [1 - q_0, 1)$. The new state vector $\tilde{X} = [\bar{K}_g, r_{kb}, A] \in \mathbb{R}_+ \times [1 - q_0, 1) \times \{A_h, A_l\}$.

As for the approximation methods for these functionals, I choose tensor product of "not-a-knot" cubic splines as used in Khan and Thomas(2003). The rest of the computation algorithm includes inner loop and outer loop. Given laws of motions of H_1 and H_2, the inner loop finds (p_u, λ, x) that satisfy no-arbitrage condition, used market clearing condition and consistency of belief. The outer loop is to update the laws of motions until converge. The last step is to check whether the computed results satisfy the equilibrium conditions. If $x(\tilde{X}) \in (0, 1)$ and $\lambda(\tilde{X}) \in (0, q_0)$ thus $p_u(\tilde{X}) < 1$ for all $\tilde{X}$, it is the equilibrium of interest.

[3]The solution to equation (2.6),(2.7),(2.8),(2.9) and (2.10) may have some points where $p_u > 1$ and $\lambda > q_0$, which does not have economic sense. In other words, the state space of $(\bar{K}_g, \bar{K}_b)$ is larger than the set of $(\bar{K}_g, \bar{K}_b)$ in equilibrium. It is okey in mathematics, but it is hard to isolate the case where the equilibrium of interest does not exist. One way to solve this problem is to use non-rectangular grid points. But the technique for multi-dimensional interpolation over such areas is under-developed. That's why this paper uses a different state space.

2.4 Characterization of Stationary Equilibrium

2.4.1 Calibration

Table A.9 summarizes baseline parameter values. I use standard parameters as much as possible and calibrate the non-standard parameters in a way consistent with relevant micro and macro level moments. The purpose is to study the cyclical implication in the presence of adverse selection with the parameterized model.

The model period is one year. The discount rate of plants β equals 0.95. The depreciation rate δ is 0.069. The curvature of production function, $\alpha = 0.592$. In the paper, the production function is in fact the profit function, which can be derived from plants' profit maximization problem with static labor choice. Thus the curvature follows the estimation in Cooper and Haltiwanger (2006).

The distribution of plant-specific productivity F is calibrated by the stationary distribution of idiosyncratic shocks in Cooper and Haltiwanger (2006). They assume the idiosyncratic shocks follow an AR(1) process and estimate the coefficients with plant-level data. The persistence coefficient is 0.885 and standard deviation for idiosyncratic innovations is 0.3. I use Tauchen's method to transform the AR(1) process into a 40-state stationary Markov process. Then use the transition matrix to compute the stationary distribution. The upper and lower bounds of plant-specific productivity are 0.2756 and 3.6280.

Parameters of aggregate uncertainty follow Eisfeldt and Rampini (2006). The high aggregate uncertainty level $A_h = 1.015$, the low aggregate uncertainty level $A_l = 0.985$. The conditional probabilities π_{hh} and π_{ll} are both 0.75. They choose the parameters to match the facts that the frequency of U.S. business cycles is four years and annual standard deviation of the logarithm of aggregate total factor productivity is 15%.

The model endogenizes the irreversibility of an investment problem. The degree of irreversibility depends on the parameterizations. The rest of parameters are the fraction of good capital in new capital market, q_0, the efficiency of bad capital ϕ and the initial capital stock k_0. I choose non-standard parameters to match some selective moments in the steady state. The first target moment is the fraction of used capital expenditure, the average of which is 7.3158%[4] according to "Annual Capital Expenditures Survey " from 1996 to 2006. The second target moment is the annual entry rate, measured as the ratio of capital stocks of entrants over total capital stock. Since there is no good estimation for the ratio in terms of capital stock[5], I use the average job creation of entrants between 1973 and 1988 in Davis, Haltiwanger and Schuh (1996) instead. The annual entry rate is 1.44%. The third target moment is the fraction of inactive plants. Cooper and Haltiwanger (2006) report that the fraction is 8.1%. Their definition of inactive plants is those whose gross investment is less than 1%. Table A.10 summarizes the calibration results. The calibrated parameters match the first two moments well. But for the third moment, owners of lemons always sell their bad capital, the negative investment is high in the model. Thus the fraction of inactive plants is low.

[4]The two period plant problem makes the fraction of used capital very high. In reality, most of capital stock of old plants are not sold in the used capital market. I therefore use the fraction of used capital expenditure from young plants to match the data moment.

[5]Becker, et al(2004)report roughly that the ratio is less than one percent. But they also point out that their data set missed observations at some boom years whcih makes the estimation lower

The aggregate state variables $\bar{K}_g$ and rK_b are both discretized into 49 equally distributed grids. The upper bound for rk_b, the ratio of aggregate bad capital, is 0.324. The lower bound is 0.3206. I also set the upper and lower bounds of aggregate good capital, $\bar{K}_g$, are 65.1 and 61.9. Plant-specific productivity as mentioned before has 40 states between 0.2756 and 3.6280.

2.4.2 The Equilibrium Used Capital Price

Graph A.2 plots the equilibrium outcomes as functions of $\bar{K}_g$ when $rK_b = 0.3215$. Graph A.3 plots the equilibrium outcomes as functions of $\bar{K}_b$ when $\bar{K}_g$ is fixed at 62.77. The solid lines stand for functions at the good state and the dotted lines represent those at the bad state. The good sate means that it has high aggregate uncertainty level.

The consistent beliefs are the same both at the high and low uncertainty level according to the right panels of Graph A.2 and A.3, which results from the approximation of the continuous distribution function. Although the support for the distribution function is $(0, \infty)$, the discrete state space has lower bound. As shown in the proof of Proposition 2.3.4, the lower threshold $\underline{a}_t$ is an increasing function of k_0. The calibrated k_0 is so small that even the smallest grid of the plant-specific productivity distribution is above the lower threshold value. That indicates no young plants sell good capital in the approximated model economy.

Graph A.2 and A.3 show that the used capital price increases with aggregate uncertainty level when fixing $(\bar{K}_g, rK_b)$. That's because the supply for used capital is inelastic while the demand is larger at the high uncertainty level. Then the used capital price is higher at the good state than that at the bad state.

In graph A.2, used capital price and consistent belief increase with $\bar{K}_g$. Since the fraction of bad capital from old plants rK_b is fixed and no young plants sell good capital, the more good capital is in the market, the higher the average quality of the used capital is. Thus the consistent belief λ increases. When the average quality is high, the no-arbitrage condition between new and used capital markets drives the used capital price up. In graph A.3, used capital price and consistent belief decreases with rK_b. Since the more bad capital is in the market, the lower the consistent belief λ is. The used capital price declines because of the interaction between new and used capital markets.

The equilibrium used capital price is very close to 1. That's because old plants have to exit and sell out their capital. Then the quality of used capital is very close to that of new capital. According to Proposition 2.3.3, the used capital price is between the ratio of average qualities and one. Thus the price is very close to one. This result doesn't mean that the affect of information friction is neglectable. If extending the life cycle of plants, we can anticipate different result. The analysis here in fact sheds light on the qualitative effect of adverse selection on investment.

2.4.3 Cyclical Properties

This section studies the cyclical property of the model. Here one time series is procyclical means that its cross-sectional correlation with aggregate uncertainty is positive. With the price functions and laws of motions, I simulate the model economy for 10000 periods. After dropping the initial 1000 periods, I compute the correlations of equilibrium outcomes with aggregate uncertainty each period. Table A.5 summarizes the cyclical results.

The consistent belief is procyclical because new capital expenditure is higher at the good state. The new capital has higher quality than used capital. Thus the average quality of old plants' capital stocks tends to be high at the good state. The quality of used capital next period is also high. When the aggregate uncertainty is persistent, old plants sell relatively more good capital at the good state. Thus the average quality of used capital is procyclical, so is consistent belief.

The used capital price is procyclical because of the interaction between new and used capital market. The no-arbitrage condition links the (relative) used capital price with average quality of used capital. No matter how the supply side change, the used capital price increases with consistent belief. Thus we can not analyze the supply and demand in an isolated used capital market.

In the model economy, aggregate uncertainty affects used capital price through two potential channels. First, high aggregate uncertainty increases demand for used capital and decreases supply of good capital from young plants. The used capital price tends to increase. Second, the decreasing sales of good capital lower the fraction of good capital in the market, so is the consistent belief. The used capital price decreases with the average quality in the market because of no-arbitrage condition. In the calibrated model, no young plants sell good capital and the supply for used capital is inelastic.

The new and used capital with different qualities just like the capital with different vintage. We can understand the the underlying story in such a way: Plants invest on vintage capital. In good time, they invest more in new capital, which has higher quality. They sell old vintage capital, which has lower quality. If the aggregate productivity shock is persistent, the average quality in the used market is high at the good state. Thus the relative used capital price is high at the good state.

Total used and new capital expenditure are pro-cyclical. That is consistent with procyclical investment in the data. The procyclical used capital price implies that the opportunity cost to sell used capital is lower in booms. This cyclical implication is consistent with the counter-cyclical capital reallocation cost in in Eisfeldt and Rampini (2006) [6]. In their model, the reallocation cost is the extra cost when selling capital across sectors. In a calibrated model, they find only countercyclical reallocation cost can match the coexistence of procyclical quantity and countercyclical benefit of capital reallocation. Although they assume the capital reallocation cost has some specific function form, they infer the cost arising from informational and contractual problems in the economy since all physical adjustment costs measured by output are procyclical. This paper constructs and calibrates a structural model where the investment faces information problems. The cyclical result of the model is the same as their findings. Thus information friction may play an important role in capital reallocation.

[6] Their measure of capital allocation is in the firm level which includes acquisition. However, the amount that their capital reallocation minuses acquisition is also procyclical.

2.4.4 Robustness

This section checks the robustness of cyclical results when varying the parameterizations of the model. The first exercise is to examine whether the persistence of aggregate uncertainty determines the magnitude of correlation between irreversibility and uncertainty. Table A.6 presents the cyclical results when varying the persistent level of uncertainty. $\pi_{ll} = \pi_{hh}$. The higher the persistence is, the higher the correlations of the consistent belief and used capital price are. This indicates that the persistence is important to understand the cyclical results of the model. The second exercise explores the robustness of the cyclical results when varying non-standard parameters. Table A.7summarizes the results . The used capital prices are all procyclical.

2.5 Conclusion

This paper studies a dynamic investment model with state-dependent irreversibility. Investment is partial irreversible because of adverse selection in the used capital market. Irreversibility is state-dependent because the equilibrium level of irreversibility is determined by plants' investment decisions, which vary with aggregate state. The higher the aggregate state is, the less irreversible the investment is. One implication of the model is that capital adjustment cost arising from lemons problem is countercyclical. This is consistent with the finding in Eisfeldt and Rampini (2006).

Chapter 3

Partial Irreversible Investment and Cyclical Capital Reallocation: the Role of Adverse Selection

3.1 Introduction

Evidence shows that capital reallocation, measured as the aggregate sales of (used) capital, is procyclical.[1] Figure A.5 plots the cyclical components of output and the aggregate Sales of Property, Plant and Equipment (SPPE) from the Compustat database, which is the proxy for the sales of used capital.[2] This graph displays a significant positive correlation between SPPE and output. It is well known that the investment in new capital is procyclical. The joint observation of procyclical investment and the sales of used capital is interesting: In a good state, the aggregate technology shock is high, thus plants invest more. But why do plants also sell more?

Evidences also shows that plants sell their capital even after age-related depreciation (Ramey and Shapiro (2001), Cooper and Haltiwanger (2006)). Veracierto (2002) studies the implication of micro-level irreversible investment on conventional business cycle statistics (aggregate consumption, investment and labor input).

[1]In this paper, the sales of capital are equivalent to the sales of installed capital or the sales of used capital.

[2]The aggregate time series is constructed from the annual firm level data. The detailed description is in Appendix C.1.

However, existing studies do not explore the aggregate implication of irreversible investment on capital reallocation. Although adverse selection and capital specificity are two main explanations for irreversible investment, previous literature does not model the underlying mechanism explicitly. And the levels of irreversibility are exogenously chosen in Veracierto (2002).

This paper studies the aggregate implication of irreversible investment on capital reallocation after establishing the consistency with evidences on selling price of used capital. In a calibrated economy, the present paper confirms the result in Veracierto (2002) that micro-level non-linearity arising from irreversible investment does not matter for the conventional business cycle statistics. However, the fluctuations of capital reallocation are very different between models with and without micro-level irreversible investment. In particular, I find that the model with endogenous irreversibility arising from adverse selection explains the cyclical movement of capital reallocation well.

In this paper, I employ adverse selection in the used capital market to explain the procyclical sales of capital. The model allows for capital goods of different qualities, which are measured by their productivity. Capital quality is unobservable to buyers. They choose to invest in new or used capital goods. Since the quality of the used capital is higher than that of the bad (low quality) capital, plants always sell all of their bad capital first, regardless of whether they are in a good state or bad state. Thus, the sales of capital do not drop significantly when the economy faces a positive aggregate shock. In addition, the model economy consists of a continuum of plants with heterogeneous technology shocks. The plants with high idiosyncratic shocks accumulate more capital at the same time. Accordingly, the sales of bad capital become higher over time. As the aggregate technology shock is persistent, the sales of capital are procyclical after combining the two effects.

In addition, plants with low idiosyncratic technology shocks may sell part of their good (high quality) capital. Selling good capital is costly since the quality of the good capital is higher than that of used capital. Like the standard partial reversible investment model, a plant's optimal investment decision follows a two-sided (S, s) policy after selling all of its bad capital. That is, given a plant's idiosyncratic shock and aggregate states, there exist two cut-off values. First, the plant sells its good capital when its capital stock is above the upper threshold. Second, the plant buys capital when its capital stock is below the lower threshold. Otherwise, the plant does not adjust its capital. When plants purchase capital, they are indifferent concerning new and used capital, because of the no-arbitrage condition between the two markets. Therefore, the model explains the procyclical sales from the supply rather than demand side.

The investment in this model is costly reversible because of the private information in the used capital market. The price of used capital, as a measure of the irreversibility of investment[3], is endogenously determined by the interaction between the new and used capital markets and the degree of adverse selection. Therefore, the used capital price also depends on aggregate states. In contrast, the level of irreversibility is assumed to be fixed across states in the previous investment liter-

[3]The irreversibility refers to the property of capital in that the resale value of capital is lower than its purchasing cost.

ature (Bloom (2007), Cooper and Haltiwanger (2006), Faig (2001), and Veracierto (2002)). So I term their models "fixed irreversibility models", where the level of irreversibility can be interpreted as the price of used capital.

The adverse selection model is calibrated to match the evidence of the price discount in selling capital and the fraction of used capital expenditure over total capital expenditure in the Annual Capital Expenditure Survey (ACES). Finally, the model successfully replicates the procyclical sales of capital in the data. Moreover, the correlation between the sales of capital and output is 75.2% of its data counterpart.

However, a decentralized economy with fixed irreversibility arising from capital specificity fails to generate the procyclical sales of capital, when using the same calibration strategy. In order to examine the capital reallocation across firms, the fixed irreversibility model is decentralized to an economy with a fixed price of used capital. [4] In particular, plants sell their capital when they encounter low technology shocks. The demand for used capital must fluctuate along with the movement of the sales of capital to clear the market. When the economy faces a common positive technology shock, all plants have higher technology levels. They would like to maintain more capital. Therefore, the sales of capital are counter cyclical in this kind of models.

The relative price of used capital is found to be procyclical in the adverse

[4]This model economy is a decentralized version of Veracierto (2002) where the plant loses a constant fraction of values when selling capital. See Appendix C.4 for details.

selection model, which concurs with the estimation results in Li (2007).[5] In the model economy, when there is a positive aggregate shock, the price of used capital first goes down, since less good capital is sold in the market. Then the price increases as the extra capital accumulated previously increases plant expectation for the price. The increased new investment also drives the price up by improving the quality of used capital. Finally, the used capital price returns to its original level together with the aggregate capital stock.

Procyclical capital reallocation is first documented in Eisfeldt and Rampini (2006) along with the countercyclical benefit of capital reallocation. In a calibrated two-sector model, they impute the countercyclical capital reallocation cost. In their model, capital reallocation includes acquisition and sales of capital. Their cost of capital reallocation is in the form of exogenous convex capital adjustment cost, while my model aims to understand the procyclical sales from the view of micro-foundation. The relation between adverse selection and investment in my paper is similar to that described in House and Leahy (2004). They study the impact of adverse selection in dynamic durable goods markets, where the durable goods last two or three periods. My paper examines the quantitative impact of adverse selection in a dynamic stochastic general equilibrium framework, which includes new and used capital markets. As well, the investment also has extensive and intensive margins in my model.

The remaining parts of the paper are organized as follows. Section 3.2

[5]See section 3.2 for details.

presents the relevant data facts. Section 3.3 describes the baseline investment model with adverse selection. Section 3.4 characterizes the properties of decision rules and the equilibrium. Section 3.5 discusses the calibration of the baseline model. Section 3.6 reports the macroeconomic implication of the baseline model and section 3.7 presents the conclusions.

3.2 Data Facts

This section documents the main data facts that the paper aims to explain. First, the amount of capital reallocation from incumbents is procyclical. Second, plants sell their capital at a discount even after age-related depreciation. The discount of selling price differs between continuing plants and exiting plants.

In this paper, capital reallocation is defined as the reallocation of productive assets across firms which involves physical movement of capital. The quantity of capital allocation equals the used capital expenditure, which emphasizes the investment decisions. The capital reallocation, or the used capital expenditure, includes the sales of capital from both incumbents and exiters. I use the annual data on Sales of Property, Plant and Equipment (SPPE) in the Compustat database as the measure for sales of capital from incumbents[6]. (See Appendix C.1 for details.)

[6]It is worth noting that, sales of property, plant and equipment from large incumbents, for example, in the Compustat database, are not equal to the used capital expenditure of large incumbents. Generally, the sales of used capital are related to the size of the firm. According to Columbia Industrial Survey, the correlation between sales of fixed assets, excluding lands, with total employment is 0.2, which is significant at one percent significance level. However, large firms invest less on used capital than small firms. As stated in Table 5, in the report of Annual Capital Expenditure survey in 1995, the new equipment expenditure for companies with five employees or more is 6.77 times of that for companies with fewer than five employees, while the amount of used equipment expenditure

Sales of property,plant and equipment[7] are procyclical. After deflating by the Consumer Price Index (CPI)[8], the correlation of SPPE with output is 0.4479 using hp-filtered log series. The correlation is 0.4537 using linear-trended log series. The two statistics are both positive and significant. Table A.8 summarizes the results. Figure A.5 plots the cyclical component of SPPE against that of GDP. The SPPE drops substantially when the GDP is significantly low.

According to the Annual Capital Expenditure Survey (ACES), the mean fraction of used capital expenditure over total capital expenditure is 7.5%. Since the survey is designed to provide broad-based statistics for both new and used capital expenditure, it includes all the domestic, private, and non-farming firms.

The rest of the data facts is related to the discount in the selling price. Ramey and Shapiro (2001) directly estimate the resale value of capital relative to replacement cost in a shrinking aerospace industry. The estimated discount is found to be $41.9\% - 83.6\%$[9] for these exiting plants. Using indirect evidence (Simulated Method of Moments), Cooper and Haltiwanger (2006) report that the discount is 98.1%[10] for continuing plants. In my model, the parameters are calibrated according to the direct evidences: the mean fraction in ACES and the discount in Ramey and Shapiro (2001). Then the endogenous price discount is compared with the

is almost the same (1.06 times) as that for companies with fewer than five employees. The data on structure expenditure also replicated similar results.

[7]Without any further notation, the SPPE means the SPPE from incumbents.

[8]Eisfeldt and Rampini (2006) studies the cyclical results for reallocation among different capital price deflators in addition to CPI. They find the results are essentially same.

[9]The result is from Table 5 in Ramey and Shapiro (2001).

[10]The result is from Table 5 in Cooper and Haltiwanger (2006) .

estimate as in Cooper and Haltiwanger (2006).

The direct evidence for the cyclical property of resale price is limited. Pulvino (1998) studies commercial aircraft transactions and finds that the resale prices are lower during recessions. Since there is currently no historical data available on the issue of used capital prices, Li (2007) uses an indirect inference procedure to estimate the cyclical component of the used capital price, assuming the price is a function of the aggregate shock. The estimation results indicate that the relative used capital price has a procyclical component, which concurs with the result in this paper.

3.3 The Model

The model economy embeds adverse selection in the used capital market into an otherwise standard real business cycle model. There is a continuum of households and a continuum of production units, which I call plants. Households own plants by holding equities. Unlike the stochastic growth model, there exist two capital markets: new and used. Plants can invest in both markets, but only disinvest in the used capital market. The only capital adjustment cost comes from partial irreversibility arising from private information in the used capital market. In this section, I first explain the information structure and quality of capital, then describe plants, households and equilibrium.

3.3.1 Information Structure and Quality of Capital

In the model, the plants produce one kind of commodity which can be used as consumption or investment. The commodity is homogenous for consumption while it has different qualities when using as investment. Once the good is installed for purposes of production, I call it capital. The capital good has two quality types: good or bad. The quality refers to the productivity of capital in production. The quality of good capital is designated a value of 1, and that of bad capital is designated a value of 0. That is, in production, the good capital can be fully used while the bad capital is useless for production[11]. Only owners know the true capital type. Thus, plants have private information about the qualities of their capital. The fraction of good capital in the output is q_0. Thus, in the economy, the quality of capital is determined in the process of production[12].

There are two kinds of capital markets in this economy: new and used. In the new capital market, plants sell their output. Since even the producers do not know the qualities of their products, the new capital market is pooled. As the new capital good has q_0 fraction of good capital, the average quality of new capital is q_0[13]. For simplicity, I assume that every unit of new capital bought from the market

[11]If the assumption is relaxed and the bad capital has a positive productivity, but less than the value of 1 which is assigned to good capital, the main results of the paper will not change. Please refer to my previous paper "State-dependent Irreversibility and Lemons Market".

[12]An alternative assumption is to let the capital deteriorate in the process of utilization with some fixed probability. Under this assumption, the quality of new goods is 1. The level of relative used capital price will change accordingly, but the properties of decision rules and equilibrium won't change.

[13]The average quality equals the weighted average of good and bad capital. $q_0 = q_0(1) + (1 - q_0)(0)$.

contains q_0 fraction of good capital. If the capital good is infinite-divisible, this assumption acts as the law of large numbers. The price of new capital is normalized to 1 in each period.

The used capital market acts like a middleman who trades with plants at an equilibrium price. In the market, sellers know the qualities of their capital. They can choose to sell good or bad capital while buyers can not observe the quality of the used capital. What they buy is a combination of good and bad capital. Both buyers and sellers are price-takers. The equilibrium price is pinned down when the demand for used capital equals the total sales of good and bad capital. In this paper, I impose the pooling equilibrium in the used capital market, and exclude any possibility of alleviating the information problem by forming contracts.

Unlike the new goods market, the quality of used capital is endogenously determined by plants' investment decisions. As buyers can not observe the qualities of capital, they make investment decisions based on their belief, which represents the anticipated fraction of good capital sold in the market. In equilibrium, the plants' belief equals the realized fraction of good capital in the market. Denote the belief as λ. In equilibrium, the average quality of used capital is then λ. Similarly to the new capital market, I assume that buyers get λ fraction of good capital per unit of used capital. The relative used capital price, $p_{u,t}$, is then determined by the interaction between plants' investment decisions and the degree of adverse selection.

In this paper, the investment is costly reversible. It has two implications: First, the selling price of capital is less than the purchasing price. Thus, plants only get part of their initial investment after depreciation, when selling their capital;

Second, at the aggregate level, the installed investment goods can not be used as consumption goods. For this reason, the model economy has a separate used capital market.

In the previous literature, the relative used capital price is fixed[14]. The new and used capital goods are perfect substitutes. Thus the fluctuation of the sales of capital has no impact on the determination of used capital price, since the demand for used capital has to change accordingly to clear the market. So the existence of a separate used capital market is trivial as long as aggregate investment is strictly positive.

3.3.2 Plants

The model economy consists of a continuum of plants. A typical plant produces its output by employing good capital stock k_g and labor n. The output can be used as consumption goods or new investment goods. When the output is used as new investment goods, it contains q_0 fraction of good capital. Each plant employs a Cobb-Douglas Decreasing Return to Scale production technology,

$$y = aAk_g^{\alpha}n^{\nu} \text{ with } \alpha \in (0,1); \nu \in (0,1); \alpha + \nu < 1$$

Here, a is the idiosyncratic technology shock while A reflects the aggregate technology shock, or stochastic total factor productivity. The two shocks are independent and multiplicative to the production procedure. The aggregate technology

[14]In a model without costly reversible investment, the relative used capital price is 1. In a model with fixed irreversibility, the selling discount of capital is a constant, which is less than 1. The relative used capital price is still 1 in a decentralized economy. See Appendix C.4 for the model.

shock, $A \in Z_A$, follows a continuous Markov process with a stationary transition function. Z_A is a compact Borel set in $\mathbb{R}_+$. The idiosyncratic shock, $a \in Z_a$, is specified as a n_a state Markov chain with a stationary transition function Q_a. $Z_a = \{a_1, a_2, ...a_{n_a}\}$. To simplify the notation, denote Z as $Z_a \times Z_A = \{z = (a, A) : a \in Z_a, A \in Z_A\}$. $\mathcal{Z}$ is the Borel subsets of Z. The transition function, $Q : Z \times \mathcal{Z} \to [0, 1]$, is monotone and has the Feller property.

In addition, the plant is subject to an exit shock before the realizations of aggregate and idiosyncratic shocks. That is, a plant has a probability of exiting the economy at the beginning of each period. Let η be the surviving rate. $\eta \in (0, 1)$. $1 - \eta$ is the exiting probability. When a plant exits the economy, it has to liquidate its capital stock in the used capital market. Because of the continuum of plants, the measure of exiting plants is $1 - \eta$ if the total measure of plants is 1. At the end of each period, the new plants enter the economy with zero capital and different initial productivity levels, which are drawn from the stationary distribution of the idiosyncratic shock F_{ss}. For tractability, the measure of total plants is fixed across time. Thus the measure of entrants is $1 - \eta$ in each period.

In every period, a plant is defined by its good capital stock, $k_g \in E_g$, bad capital stock, $k_b \in E_b$, and idiosyncratic technology shock $a \in Z_a$. $E_g = [0, \bar{k}_g]$, $E_b = [0, \bar{k}_g]$. $\bar{k}_g$ and $\bar{k}_b$ are described explicitly in Appendix C.2. At the beginning of each period, a plant first observes its exit shock. If exiting, the plant has to sell its good and bad capital stocks. Otherwise, the plant observes its idiosyncratic shock and aggregate states. The incumbent then continues production and makes labor and investment decisions. At the same time, new plants enter the market with zero capital and initial idiosyncratic productivity draw. Figure A.6 plots the timing and evolution of the economy.

When an exiting plant liquidates its capital, it has an extra selling discount due to capital specificity. Denote p_c as the fraction of capital stock that can be sold in the used capital market. Since the used capital price is p_u, the liquidating value is $p_u p_c$ per unit of capital stock, which includes both good and bad capital.

Plants choose to purchase capital from new and used goods markets. Denote i_n as the investment in new capital goods. $i_n \geq 0$. Denote i_u as the investment in used capital goods. $i_u \geq 0$. Plants choose to sell good capital $i_g \in E_{ig} = [0, k_g(1-\delta)]$, or to sell bad capital $i_b \in E_{ib} = [0, k_b(1-\delta)]$. In the economy, output, capital and consumption grow at rate $r - 1$ along the balanced growth path. r is the gross growth rate. Therefore, all the variables are expressed in terms of efficient units. All the variables are deflated by the level of labor augmenting technology. The evolutions of the plant's good and bad capital stocks are as follows[15]

$$rk'_g = (1-\delta)k_g + i_n q_0 + i_u \lambda - i_g; \;\; rk'_b = k_b(1-\delta) + i_n(1-q_0) + i_u(1-\lambda) - i_b \quad (3.1)$$

$\delta \in (0, 1)$ is the depreciation rate of capital. Let $y = (k'_g, k'_b, i_g, i_b) \in Y$ be the control variables in the dynamic programming problem, $Y = E_g \times E_b \times E_{ig} \times E_{ib}$. Then[16]

$$\begin{aligned} i_n &= \frac{1}{q_0 - \lambda}(rk'_g(1-\lambda) - rk'_b\lambda - (k_g(1-\lambda) - k_b\lambda)(1-\delta) + i_g(1-\lambda) - i_b\lambda) \\ i_u &= \frac{1}{\lambda - q_0}(rk'_g(1-q_0) - rk'_b q_0 - (k_g(1-q_0) - k_b q_0)(1-\delta) + i_g(1-q_0) - i_b q_0) \end{aligned}$$

[15] The primes stand for values one period ahead.
[16] In section 3.4, I show that $\lambda < q_0$ in equilibrium.

The aggregate states include aggregate technology shock $A \in Z_A$ and the distribution of plants over (k_g, k_b, a). Let $s = (k_g, k_b, a) \in S$. Denote $S = E_g \times E_b \times Z_a$ by the product space for individual state variables. Let $\mathcal{S}$ be the $\sigma-$algebra of S. The distribution of plants over the triple is then characterized by a Borel probability measure μ on $\mathcal{S}$. Denote $\mathcal{D}$ as the set of distribution functions on $\mathcal{S}$. $\mu \in \mathcal{D}$. The evolution of this distribution is characterized by a mapping H, which is described explicitly in section 3.4. Let $X = S \times Z_a \times \mathcal{D}$ be the product space of state variables.

The following correspondence describes the constraints on the control variables:

$$\Gamma(s, A, \mu) = \{y \in Y :, i_n \geq 0; i_u \geq 0\} \text{ for all } (s, A, \mu) \in X.$$

The incumbent makes employment and investment decisions after observing its idiosyncratic shock and the aggregate states. The plant's labor decision is static. So I first derive the explicit form of labor demand from the one-period profit maximization problem.

$$n(s, A, \mu) = (\frac{\nu a A k_g^\alpha}{w})^{\frac{1}{\nu-1}}; \quad f(k_g, a, A) = (aAk_g^\alpha)^{\frac{1}{1-\nu}} w^{\frac{\nu}{1-\nu}} (\nu^{\frac{\nu}{1-\nu}} - \nu^{\frac{1}{1-\nu}})$$

w is the wage in the model. The one period return function, $F : X \times Y \to \mathbb{R}$, is

$$F(k_g, k_b, a, A, \mu, y) = f(k_g, a, A) - i_n - p_u i_u + p_u i_g + p_u i_b$$

i_n and i_u are defined as above. The plant aims to maximize its expected discounted value. As the plant is owned by households, it discounts the future value with a stochastic discount factor, $d(A, \mu) = \beta \frac{Ep(A', \mu')}{p(A, \mu)}$. $p(A, \mu)$ is the marginal utility of consumption. Now the investment problem can be formulated in terms of utilities. The following is the plant's optimization problem,

$$v(k_g, k_b, a; A, \mu) = max_{y \in \Gamma(s,A,\mu)} F(k_g, k_b, a, A;)p(A, \mu)$$
$$+ \beta\eta \int v(k_g', k_b', a', A', \mu')Q(z, dz') + \beta(1-\eta)p_c \int (k_g' + k_b')(1-\delta)p_u' p' Q(z, dz') \tag{3.2}$$

In the right-hand side of equation (3.2), the second term states the discounted present value as an incumbent next period, and the third term is the discounted present value as an exiting plant next period. β is the discount factor of a representative household. Given the used capital price $p_u(A, \mu)$, the intertemporal price $p(A, \mu)$ and the law of motion $\mu' = H(\mu, A)$, the plant's labor demand is $n(k_g, k_b, a; A, \mu)$. The next period's capital stocks are $k_g'(k_g, k_b, a; A, \mu)$ and $k_b'(k_g, k_b, a; A, \mu)$. The sales of good and bad capital are $i_g(k_g, k_b, a; A, \mu)$ and $i_b(k_g, k_b, a; A, \mu)$. The purchases of new and used capital are $i_n(k_g, k_b, a; A, \mu)$ and $i_u(k_g, k_b, a; A, \mu)$. In the Appendix C.2, I prove the uniqueness of the fixed point in the dynamic programming problem. Further in section 3.4, I simplify the problem and characterize the properties of decision rules.

3.3.3 Households

In the model, households are identical. They own the plants and accumulate dividends. In addition, the asset market is complete. Thus, I only model the static maximization problem of a representative household, and focus on the first order conditions which determine the equilibrium wage and the intertemporal price. The representative has the utility function $U(C, N^s) = logC + sl(1 - N^s)$ The intertemporal price, $p(A, \mu)$, is defined as the marginal utility of the household. The wage can be pinned down from the first order condition with respect to labor choice:

$$p(\mu, A) = U_C(C, N^s); \;\; w(\mu, A) = -\frac{U_N(C, N^s)}{U_C(C, N^s)} = \frac{sl}{p(A, \mu)}$$

3.3.4 Recursive Competitive Equilibrium

Three markets exist in the model economy: used capital market, new goods (capital) market and the labor market. The incumbents are the potential buyers and sellers in the used capital market. The exiting plants have to sell their capital in the used capital market. Thus the corresponding market clearing condition is:

$$\begin{aligned} \eta \int i_u(s; A, \mu) d\mu = \eta \int i_g(s; A, \mu) d\mu + \eta \int i_b(s; A, \mu) d\mu \\ + (1 - \eta) p_c \int (k_g + k_b)(1 - \delta) d\mu \end{aligned} \tag{3.3}$$

The left-hand side of equation (3.3) is the total demand for used capital. The surviving rate η in the equation indicates that only incumbents demand used capital. The first term on the right-hand side of equation (3.3) is the sales of good capital from incumbents. The second term is the sales of bad capital from incumbents. The last term is the total sales of capital from the exiting plants. Since the exit rate is exogenous, the sales in the last term are a fraction of total capital stock. New goods can be used as consumption and investment. The new goods market clearing condition and labor market clearing conditions are:

$$C(\mu, A) + \eta \int i_n(s; A, \mu) d\mu \quad = \quad \eta \int f(k_g, a, A) d\mu \tag{3.4}$$

$$N^s(\mu, A) \quad = \quad \eta \int n(s; A, \mu) d\mu \tag{3.5}$$

In equilibrium, the plant's belief about the quality of used capital is consistent with the realized fraction of good capital sold in the used capital market. The consistency belief condition is

$$\lambda = \frac{\eta \int i_g(s; A, \mu) d\mu + (1 - \eta) p_c \int (1 - \delta) k_g(s; A, \mu) d\mu}{\eta \int i_u(s; A, \mu) d\mu} \tag{3.6}$$

Definition 3. *A recursive competitive equilibrium for the economy* $\{\beta, \delta, \eta, Q, sl, \alpha, \nu, p_c\}$ *consists of: (a) Plant's policy functions,* i_n*,* i_u*,* i_g*,* i_b*,* k'_g*,* k'_b*,* n*, and value function* v*; (b) Household's consumption* C *and labor supply decision* N^s*; (c) The belief* λ*, equilibrium wage* w*, and the intertemporal price* p*; (d) The law of motion of the distribution* H*. Such that, 1. Given* w *and* p*,* C*, and* N^s *maximize the representative household's utility; 2. Given* w*,* p*,* λ*,* p_u*, and* H*,* u *solves (3.2) and the plant's decision rules are* i_n*,* i_u*,* i_g*,* i_b*,* k'_g*,* k'_b*, and* n*. 3.* w*,* p*,* p_u*, and* λ *satisfy labor market clearing condition (3.5), new good market clearing condition (3.4) and used good market clearing condition (3.5), given the decision rules of the household and the plant ; 4. The consistent belief* λ *satisfies condition (3.6); 5. The law of motion of the cross-sectional distribution,* H*, is generated by the plant's decision rules, aggregate and idiosyncratic shocks.*

3.4 Characterization of Recursive Equilibrium

In this section, I describe the properties of the equilibrium belief and used capital price, and characterize the plant's optimal investment policy. Since in the real world, the new investment, which is part of GDP, and used capital investment, which reflects capital reallocation, are always positive, the rest of this paper focuses on the equilibrium where both new and used capital investment are strictly positive. After employing the properties of the belief and used capital price, the dynamic programming problem with three individual state variables and four control variables is simplified to a problem with only one endogenous individual state variable.

3.4.1 Aggregate Capital Stocks

At the initial period of the economy, there is no used capital in the economy. Thus, the ratio of good capital in total capital stock is q_0 at the initial period. Define the aggregate good and bad capital stocks as followings:

$$K_g = \int k_g(k_g, k_b, a; A, \mu) d\mu; \quad K_b = \int k_b(k_g, k_b, a; A, \mu) d\mu.$$

The aggregate new and used capital investments are defined as:

$$I_n = \eta \int i_n(k_g, k_b, a; A, \mu) d\mu; \quad I_u = \eta \int i_u(k_g, k_b, a; A, \mu) d\mu.$$

The aggregate sales of good and bad capital goods are specified as:

$$I_g = \eta \int i_g(k_g, k_b, a; A, \mu) d\mu + \int (1-\eta)(1-\delta) p_c \bar{K}_g d\mu; \tag{3.7}$$

$$I_b = \eta \int i_b(k_g, k_b, a; A, \mu) d\mu + \int (1-\eta)(1-\delta) p_c \bar{K}_b d\mu. \tag{3.8}$$

In the economy, the capital stock at the beginning of next period is determined by the investment decisions of incumbents. The laws of motions of aggregate good and bad capital stocks are characterized as follows:

$$rK'_g = \eta K_g(1-\delta) + I_n q_0 + I_u \lambda - I_g \tag{3.9}$$

$$rK'_b = \eta K_b(1-\delta) + I_n(1-q_0) + I_u(1-\lambda) - I_b \tag{3.10}$$

The laws of motions can be rewritten as:

$$rK'_g = K_g(1-\delta)(\eta + (1-\eta)p_c) + I_n q_0 \tag{3.11}$$

$$rK'_b = K_b(1-\delta)(\eta + (1-\eta)p_c) + I_n(1-q_0) \tag{3.12}$$

The extra discount from liquidating increases the effective depreciation rate. Since the fraction of good capital stock in the economy is q_0 in the initial period, the fraction is the same in all periods. The fixed ratio helps to reduce the number of aggregate state variables when computing the approximate laws of motions.

Lemma 2. $\frac{K_{g,t}}{K_{g,t}+K_{b,t}} = q_0, \qquad \forall\ t > 0.$

3.4.2 The Equilibrium Belief and Price

The equilibrium used capital price $p_{u,t}$ is less than or equal to 1. Otherwise, given any investment decision, the plant can earn positive profit by purchasing new capital and selling it in the used capital market. Thus, optimal investment decisions do not exist. So this paper only focuses on the equilibrium with $p_{u,t} \leq 1$.

In the model economy, the exit rate is exogenous. As a result, a positive amount of good capital is always in the used capital market, since the economy consists of a continuum of plants. Therefore, the consistent belief λ_t is strictly positive. Next, I analyze the equilibrium by assuming $\lambda_t < q_0$. Later I show that the consistent belief has to be less than q_0 in the equilibrium. The plants take the aggregate laws of motions as given when making investment decisions.

Proposition 3.4.1. *The problem (3.2) has a unique fixed point $v \in C(X)$.*

To have a well-defined solution, I assume that the intertemporal price function $p(A, \mu)$ and the used capital price $p_u(A, \mu)$ are continuous and bounded functions on $Z_a \times \mathcal{D}$. The proof in Appendix C.2 is standard by applying Contraction Mapping Theorem and the Maximum Theorem.

Proposition 3.4.2. $\forall\ x \in X$, $i_u(x) \cdot i_g(x) = 0$. $i_b(x) = k_b(1 - \delta)$.

In an optimal investment policy, plants sell all their bad capital. If they sell a positive amount of good capital, their purchase of used capital must be zero. Similarly, when they purchase used capital, the sales of good capital should be zero. This is intuitive since the quality of used capital is less than the quality of good capital, but higher than that of bad capital. If a plant has some amount of bad capital, it can sell the bad capital and buy the same amount of used capital without any extra cost. Even though the total amount of capital stock does not change, the quality of the capital improves. A similar story applies to the choice between the sales of good capital and the purchases of used capital. In Appendix C.2, I rewrite the problem by using total capital stock as one of the state variables to prove the proposition.

The proposition 3.4.2 still holds when $\lambda \geq q_0$. Now plants have even higher incentive to sell their bad capital in the used capital market. On the other hand, plants will not sell all their good capital because of Inada condition. Thus, in equilibrium, the consistent belief is always less than q_0.

Proposition 3.4.3. *The equilibrium belief* $\lambda_t \in (0, q_0)$.

Since plants sell all their bad capital, the marginal benefit of bad capital is a constant over individual states. This argument leads to the following proposition. The marginal product of bad capital is in terms of utility in this proposition.

Lemma 3. $v(k_g, k_b, a; A, \mu)$ is partially differentiable with respect to $k_b \in (0, \bar{k}_b)$ for every $(a, A, \mu) \in Z \times \mathcal{D}$. Further, $v_2(k_g, k_b, a; A, \mu) = pp_u(1-\delta)$.

The fixed marginal benefit of bad capital across individual states implies that plants choose new or used capital investment concurrently. Plants have two ways to improve bad capital: to invest in new or used capital markets. The purchasing cost for the new or used capital is the same across individual plants. Thus, if the relative cost for new capital is lower, plants will all invest in new capital. Otherwise they all choose used capital. Since this paper only examines the equilibrium with strictly positive new and used capital investments, the following proposition presents a condition where new and used capital investments can coexist.

Proposition 3.4.4. *[no-arbitrage condition]*

$$p_u = \frac{rp\lambda + (q_0 - \lambda)((1-\eta)p_c + \eta)\beta(1-\delta)E(p'_u p')}{rpq_0} \tag{3.13}$$

Under no-arbitrage condition, the return on investment in new capital equals the return on used capital. In other words, new and used capital are perfect substitutes. Denote MR_g[17] as the marginal benefit of increasing one unit of good capital stock. When increasing one unit of new capital, its cost is rp units in terms of efficiency and utility. The marginal benefit from good capital is $q_0 MR_g$, and the marginal benefit from bad capital is $(1-q_0)MR_b$. The cost for new capital is 1. When a plant buys one unit of used capital, or rp unit in terms of efficient units and utility, the marginal benefit from good capital is λMR_g, and the marginal benefit from bad capital is $(1-\lambda)MR_b$. The cost for used capital is p_u. MR_g and MR_b can be derived from Khan-Tucker condition. Now the return of new capital equals the return of used capital. $q_0 MR_g + (1-q_0)MR_b = \frac{1}{p_u}(\lambda MR_g + (1-\lambda)MR_b)$.

Proposition 3.4.5. *The equilibrium used capital price $p_{u,t} \in (\frac{\lambda_t}{q_0}, 1)$.*

According to 3.13, the used capital price is related to consistent belief λ, stochastic discount factor $\frac{\beta E p'}{p}$ and the expected used capital price Ep_u. $q_0 < \lambda$,

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[17] The value function is differentiable almost everywhere since it is monotonic, thus it is proper. For those non-differentiable points, the marginal product of good capital is in fact the subdifferential at the optimal solution. As a closed proper concave function with a convex feasible set, the maximum of the problem achieves when satisfies Kuhn-Tucker conditions. When the plant buys capital, the Lagrangian Multipliers for $i_n \geq 0$ and $i_u \geq 0$ are zero. The corresponding Khan-Tucker conditions are

$$-\frac{r(1-\lambda-p_u(1-q_0))}{q_0-\lambda}p+\beta(1-\eta)(1-\delta)E(p_u'p')p_c+\beta\eta\int\partial u_1+m_1\frac{r(1-\lambda)}{q_0-\lambda}+m_2\frac{\lambda(1-q_0)}{\lambda-q_0}=0$$

$$\frac{r(\lambda-q_0p_u)}{q_0-\lambda}p+\beta(1-\eta)(1-\delta)Ep_u'p'p_c+\beta\eta\int\partial u_2-m_1\frac{r\lambda}{q_0-\lambda}-m_2\frac{rq_0}{q_0-\lambda}=0$$

The marginal benefit of the good capital is $MR_g = \beta(1-\eta)(1-\delta)E(p_u'p')p_c + \beta\eta\int\partial u_1$. The marginal benefit of the bad capital is $MR_b = \beta(1-\eta)(1-\delta)Ep_u'p'p_c + \beta\eta\int\partial u_2$.

thus $p_u > \frac{\lambda}{q_0}$. The equilibrium used capital price is less than 1 since the stochastic discount factor $\beta\frac{Ep'}{p} < 1$.

3.4.3 Simplified Version of Recursive Equilibrium

In this section I use the properties of equilibrium to simplify the dynamic programming problem. Since the new and used capital are perfect substitutes, the policy correspondence is not single-valued. I then construct an equilibrium where every plant invests x fraction of efficient units in used capital, when buying capital. Denote i_0 be the investment in good capital, $i_0 = rk_g' - k_g(1-\delta)$.

$$i_u = \frac{x}{\lambda}(rk_g' - k_g(1-\delta)); \;\; i_n = \frac{1-x}{q_0}(rk_g' - k_g(1-\delta))$$

The bad capital stock next period k_b' is $\frac{1}{r}(\frac{x}{\lambda}(1-\lambda) + \frac{1-x}{q_0}(1-q_0))i_0$, which does not depend on k_b'. Denote $X_4 = E_g \times Z \times \mathcal{D}$, the following lemma shows that the value function can be expressed as the sum of two separate functions.

Lemma 4. $v(k_g, k_b, a; A, \mu) = u(k_g, a; A, \mu) + pp_u(1-\delta)k_b$. $u(x) \in C(X_4)$ for every $x \in X_4$.

Based on Lemma 4, I focus on the function u to solve the plants' decision rules. The new one period return function $F : X_4 \to \mathbb{R}$ is:

$$\begin{aligned} F(k_g, k_g', a; A, \mu) &= f(k_g, a, A)p + p_1(rk_g' - k_g(1-\delta))\mathbf{1}_{rk_g' > k_g(1-\delta)} \\ &+ p_u(k_g(1-\delta) - rk_g')\mathbf{1}_{rk_g' \leq k_g(1-\delta)} + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p') \end{aligned} \tag{3.14}$$

where $p_1 = \frac{1}{rq_0}(\beta(1-\delta)(\eta + (1-\eta)p_c)E(p_u'p')(1-q_0) - rp)$. $p_1 < 0$ which follows from $p_u \leq 1$ and the no-arbitrage condition. p_1 is like the effective price for good capital. By applying the no-arbitrage condition, $-p_1 > p_u$. The investment in the good capital is still costly reversible. The simplified recursive problem (FE3) is as follows:

$$u(k_g, a; A, \mu) = \max_{k'_g \in [0, \bar{k}_g]} F(k_g, k'_g, a; A, \mu) + \beta\eta \int u(k'_g, a', A', \mu') Q(z, dz') \quad (3.15)$$

The policy correspondence $G_1 : X_4 \to E_g$ is as follows:

$$\begin{aligned} G_1(x) = &\{k'_g \in E_g : u(x) = F(k_g, k'_g, a; A, mu) + \beta\eta \int u(x') Q(z, dz')\} \\ &\text{for all } \ x \in X_4 \end{aligned}$$

Proposition 3.4.6. *Problem 3.15 has a unique fixed point $v \in C(X_4)$. For every $x \in C(X_4)$, $u(x) : X_4 \to \mathbb{R}$ is strictly increasing and strictly concave in the first element k_g. The policy correspondence $G_1(x) : X_4 \to E_g$ is a continuous, single-valued function.*

Since plants' investment decisions only depend on $(k_g, a; A, \mu)$, most of aggregate variables in the market clearing conditions and the consistency condition can be obtained without knowing the information about bad capital. Although the aggregate bad capital stock is present in these conditions, the amount of total bad capital stock can be derived from Lemma 2. So I only focus on the distribution over (k_g, a). Denote $S_1 = E_g \times E_a$ by the product space of individual state variables. Let $\mathcal{S}_1$ be the σ-algebra of S_1. The distribution of plants is characterized by a Borel probability measure μ_1 on $\mathcal{S}_1$. Denote $\mathcal{D}_1$ as the set of distribution functions on $\mathcal{S}_1$. $\mu_1 \in \mathcal{D}_1$. The evolution of the distribution is described by a mapping

H_1. When there is no aggregate technology shock, the model has a unique invariant distribution over the pair (k_g, a).[18] The following equations represent the used goods market clearing condition, the new goods market clearing condition, the labor market clearing condition, the consistency of belief, and consistency of laws of motions.

$$
\begin{aligned}
&\frac{x}{\lambda}\int (rh_g(k_g,a;A,\mu_1) - k_g(1-\delta))\mathbf{1}_{rh_g(k_g,a;A,\mu_1)>k_g(1-\delta)}d\mu_1 \\
&= \int (k_g(1-\delta) - rh_g(k_g,a;A,\mu_1))\mathbf{1}_{k_g(1-\delta)\geq rh_g(k_g,a;A,\mu_1)}d\mu_1 \\
&+ (1-\delta)((1-\eta)p_c + (\eta + (1-\eta)p_c)\frac{1-q_0}{q_0})\int k_g d\mu_1
\end{aligned}
$$

$$
\begin{aligned}
C(A,\mu_1) =& \eta \int aAk_g^\alpha n(k_g,a;A,\mu_1)^\nu d\mu_1 \\
&- \frac{1-x}{q_0}\int (rh_g(k_g,a;A,\mu_1) - k_g(1-\delta))\mathbf{1}_{rh_g\geq k_g(1-\delta)}d\mu_1
\end{aligned}
$$

$$
N^h(A,\mu_1) = \int N(k_g,a;A,\mu_1)d\mu_1
$$

$$
\lambda_t = \frac{\int (k_g(1-\delta) - rh_g(k_g,a;A,\mu_1))\mathbf{1}_{k_g(1-\delta)\geq rh_g}d\mu_1 + (1-\eta)(1-\delta)p_c\int k_g d\mu_1}{\frac{x}{\lambda}\int (rh_g(k_g,a;A,\mu_1) - k_g(1-\delta))\mathbf{1}_{rh_g(k_g,a;A,\mu_1)>k_g(1-\delta)}}
$$

$$
\mu_1' = H_1(A,\mu_1)
$$

Definition 4. *A recursive competitive equilibrium for the economy* $\{\beta,\delta,\eta,Q,sl,\alpha,\nu,p_c\}$ *with strict positive demands for new and used capital goods is a set of functions* $(p_u,p,x,\lambda,w,H_1,u,h_g,N^s,C,n)$ *such that: 1)* (u,h_g,n) *satisfy plant's maximization problem (3.16) given* (p_u,λ,p,x,w)*; 2)* (C,N^s) *solve household's problem given* (w,p)*; 3) Used capital market clears; 4) New goods market clears;*

[18] The full proof is available under request.

5) Belief is consistent; 6) No arbitrage condition (3.13) is satisfied; 7) Consistency of laws of motion is satisfied.

The law of motion for the distribution over the pair (k_g, a) is given by:

$$\mu_1'(E_k \times E_T; A, \mu) = \eta \int_{Z_a} \left[\int_{Z_a \times E_g} \mathbf{1}_{E_k}(h_g(k_g, a; A, \mu_1)\mu_1(d_{a\times k_g})) \right] \mathbf{1}_{E_T}(a') \\ Q_a(a, da') + (1-\eta)\mathbf{1}_{E_k}(0) \int_{Z_a} \mathbf{1}_{E_T}(a') F_{ss}(d_{s'}) \tag{3.16}$$

Here, $\mu_1'(E_k \times E_T; A, \mu)$ describes the measure of plants with $k_g' \in E_k$ and $a' \in E_T$ next period, given aggregate state $(A, \mu_1) \in Z_a \times \mathcal{D}_1$. The first term is the measure of plants who are incumbents. They transit from the current shocks to the shock a' and choose next period good capital stock $k_g' \in E_k$. The second term is the measure of plants who are entrants. When $0 \in E_k$, the second term is the measure with initial productivity draw $a' \in Z_a$.

3.4.4 Two-sided (s, S) Rules

After selling all their low quality capital, the plant's optimal policy follows a two-sided (s, S) rule. That is, given a plant's idiosyncratic shock and aggregate states, there exist two threshold values of capital stock: firstly, when the capital stock is above the upper threshold, the plant sells its good capital; secondly, when the capital stock is below the lower threshold, the plant buys capital; otherwise, the plant does not adjust its capital. The following proposition summarizes the policy.

Proposition 3.4.7. *[two-sided (s, S) rules] For every $(a, A, \mu_1) \in Z \times \mathcal{D}_1$, there exist two thresholds $x_1(a, A, \mu_1) < x_2(a, A, \mu_1)$ such that: first, $k_g' = k_1^*(a, A, \mu_1)$*

when $k_g < x_1(a, A, \mu_1)$*; second,* $k'_g = k^*_2(a, A, \mu_1)$ *when* $k_g > x_2(a, A, \mu)$*; otherwise* $k'_g = \frac{1}{r}k_g(1-\delta)$. $k^*_1 = \frac{1}{r}x_1(a, A, \mu_1)(1-\delta)$. $k^*_2 = \frac{1}{r}x_2(a, A, \mu_1)(1-\delta)$.

After deciding to buy or sell capital, the plant's optimal capital stock next period does not depend on pre-determined capital stock k_g. For $x \in X_4$, I define $k^*_1(x)$ and $k^*_2(x)$ as follows

$$
\begin{aligned}
k^*_1(x) &= \underset{k'_g \in E_g}{argmax}\ p_1 r k'_g + \beta\eta \int u(x')Q(z, dz') + \beta(1-\eta)(1-\delta)p_c k'_g E(p'_u p') \\
k^*_2(x) &= \underset{k'_g \in E_g}{argmax}\ -p_u r k'_g + \beta\eta \int u(x')Q(z, dz') + \beta(1-\eta)(1-\delta)p_c k'_g E(p'_u p')
\end{aligned}
$$

Since the value function is strictly concave, the target levels are unique. Define $x_1(x) = \frac{r}{1-\delta}k^*_1$, the cutoff value in which plants are indifferent between buying or inaction; $x_2(x) = \frac{r}{1-\delta}k^*_2$, the cutoff value in which plants are indifferent between selling or inaction.

The two-sided (s, S) policy is standard for the partial reversible investment. In this paper, the used capital price is determined in equilibrium. Thus the fluctuation of used capital price has an impact on the cutoff vales, which is different from the fixed irreversibility model.

3.5 Computation

3.5.1 Solution

The dynamic programming problem in (3.15) is hard to compute directly since the cross-sectional distribution μ_1 is infinite-dimensional. The solution is to assume that the plant makes decisions based on limited information, the mean of aggregate capital stock rather than the distribution μ_1. In this paper, I apply the two steps recursive procedure developed in Krusell and Smith (1998), Kahn and Thomas (2008), and Bachman, Caballero and Engel (2008). The procedure includes an inner loop and outer loop. Given the approximate laws of motions, I first solve the plant's optimization problem in the inner loop, then I use the policy functions to simulate the evolution of the economy, and update the approximate laws of motions. This procedure ends when the metric between two successive laws of motions, and the prediction error are both less than the stopping criteria.

The model economy has three kinds of approximate laws of motions: aggregate capital stock $\bar{K}_g$, the relative used capital price p_u, and the intertemporal price p. I choose log-linear function forms from the previous literature. Since the aggregate shock is continuous, each approximate law of motions has one regression equation[19]:

$$logK_g' = a_k + b_k logK_g + c_k logA \quad (3.17)$$

$$logp_u = a_u + b_u logK_g + c_u logA \quad (3.18)$$

$$logp = a_p + b_p logK_g + c_p logA \quad (3.19)$$

where K_g is the current aggregate good capital stock. The aggregate bad capital stock is not in the approximate functions since the ratio between aggregate good and bad capital is fixed across time. The equilibrium wage is a function of intertemporal price, thus I do not specify its rule here. Similarly, given the used capital price and

[19]In Krusell and Smith (1998) and Kahn and Thomas (2008), the aggregate shock is discrete, thus they have more than one equation for each law of motions.

the above approximate laws of motions, the consistent belief can be derived from the no-arbitrage condition. I also choose x, the fraction of efficient investment in used capital, such that the used capital market clears. Thus both x and λ are not specified here.

The inner loop begins with an initial guess of approximate laws of motions. For equation (3.17), (3.19), I use the estimates from the frictionless case. For equation (3.18), I choose a_u as the logarithm of the used capital price in the steady state, and b_u and c_u equal zero. In the inner loop, I solve the problem (3.15) after substituting K_g for μ_1 with value function iteration and Howard improvement algorithm. The aggregate shock is continuous, thus I use seven nodes Gauss-Hermite Quadrature to compute the conditional mean. (See Appendix C.5 for more details.)

The two-sided (s, S) policy reduces the computation load significantly. The two target capital levels k_1^* and k_2^* are independent of the current good capital holdings. Thus, I can compute the two target levels, and the two threshold values for each state (a, A, K_g) separately. Then I compare the value of inaction with these threshold values for each grid to determine the optimal investment decisions.

The outer loop simulates the economy and update the approximate laws of motions. First, I simulate a sequence of aggregate technology shock. Second, the economy begins with an exogenous capital distribution. The initial distribution for idiosyncratic shocks is its stationary distribution. Third,given the policy functions last period, the current p and p_u clear the new goods market and satisfy the consistent belief condition when fixing the future expectations. In other words, I use the given approximate laws of motions to compute the expected values of p_u and p. But the equilibrium conditions must hold under the current prices. Again, x is chosen to clear the used capital market, and λ comes from the no-arbitrage condition. To find the equilibrium, it is only necessary to compute two equations for two unknowns. The labor market clears naturally. Now the distribution for (k_g, a) can be derived with the equilibrium prices, decisions, and the previous distribution.

This procedure now generates a panel of p_t, $p_{u,t}$ and $K_{g,t}$. The last step of the outer loop is to find new approximate laws of motions with an OLS regression. When the distance between two sets of coefficients is small enough, the procedure is terminated. The next step is to check whether the R^2 is high enough. In this paper, the implied R-squares are higher than 0.99, this means there are no big mistakes when using the approximate rules.

3.5.2 Calibration

To evaluate the aggregate implication of endogenous irreversibility, I compute and calibrate three models: the baseline model with adverse selection, the fixed irreversibility model, and the frictionless model. In the fixed irreversibility model, the degree of irreversibility is a constant across time and state. Further, the fixed irreversibility arises from capital specificity. That is, $(1 - p_u)$ fraction of capital can not be re-utilized by other plants. Appendix C.4 describes the model in detail. The frictionless model is just the baseline model with $q_0 = 1$, i.e., no bad capital in the economy.

The model period is one year. Table A.9 summarizes the common parameters among the three models. The discount rate, β, is 0.977, which implies the annual interest rate is 4%. The growth rate r is set to be 1.016. The depreciation rate, δ, is chosen to match an average investment-capital ratio of 10.4$\%$. The parameters for preference and technology come from the calibration in Kahn and Thomas (2008). The capital share, α, is 0.256. The labor share, ν, is 0.64. The sl in the utility function is 2.4. The exogenous exit rate, η is selected to match capital destruction from exiting firms of 1% in Becker, et al. (2004).

The aggregate shock is specified as an AR(1) process in logs, in which the innovation v_z has zero mean and variance σ_z^2.

$$logA' = \rho_{agg} logA + v_z' \text{ with } v_z' \sim N(0, \sigma_z^2). \tag{3.20}$$

This is an approximation of the aggregate shock in the model since Z_A is compact. The values of ρ_{agg} and σ_z come from Kahn and Thomas (2008). They estimate the values from Solow residuals. The state space for aggregate shock has 11 grid points. I employ seven nodes Gausee-Hermite Quadrature to compute the conditional mean.

In specifying the idiosyncratic shock, I first assume a continuous shock following an AR(1) process in logs:

$$loga' = \rho_{idio} loga + v_a' \text{ with } v_a' \sim N(0, \sigma_a^2). \tag{3.21}$$

Then I discretize the process using the method from Tauchen (1986), and determine the set of idiosyncratic shocks Z_a and the transition matrix, which restricts the evolution of idiosyncratic shocks. Both ρ_{idio} and σ_z are set equal to the values in Kahn and Thomas (2008).

The rest of the parameters in the baseline model are the fraction of good capital in new output, q_0, and the extra discount when exiting firms sell their capital, p_c. These parameters are selected to match the aggregate data moments. According to Annual Capital Expenditure Survey (ACES), the mean fraction of used capital expenditure over total capital expenditure is 7.5% between 1995 to 2006. According to Ramey and Shapiro (2001), the discount in the selling price of capital is between 41.9% and 83.66% for aerospace plants that closed in the 1990s. The two moments are the target of the calibration exercise. In particular, the midpoint 0.6275 is used as the discount of the selling price. In the fixed irreversibility model, the rest of parameters are p_u^f and p_c. The calibration strategy and target moments are the same as in the baseline model. In the frictionless model, I set p_{cu}, the selling price discount for the exiting plants , 0.6275.

The calibration procedure is similar to the simulated method of moments with identity matrix. To find the parameters which replicate the evidences, I first compute the model with aggregate uncertainty with an initial guess of parameters. After finding the approximate laws of motions, I calculate the time-averaged fraction of used capital expenditure and the discount for exiting plants. Then the simulated moments is compared with data moments and the guess is updated. A good starting point is the calibration values from the model without aggregate shocks. The moments in the model without aggregate shocks are close to the time-averaged moments in the models with aggregate shocks. Given the calibration results from the model without aggregate shocks, I compute the model with aggregate shocks and use the approximate laws of motions as an initial guess for the later computation.

Tables A.10 and A.11 present the main calibration results. q_0 is 0.9972. Even though the fraction of bad capital is low in new output, the endogenous used capital price is about 0.95, which is close to the estimation in Cooper and Haltiwanger(2006), 0.981. The calibration result for fixed irreversibility case is also good. The figure in the fixed irreversibility model is 0.9824. The numerical experiments later show that the fixed irreversibility model fails to match the procyclical capital reallocation in data. It is worth noting that the fraction of used capital expenditure in the frictionless model is about 28.68%, much higher than that in the real data.

3.6 Results

Having established the consistencies of the fixed irreversibility model and the baseline model with time-averages of macro and micro evidences, I now examine the aggregate implications between these two models, and the frictionless model. When relative used capital price, is endogenously determined to clear the markets for new goods, used goods and labor, along with the fluctuations of interest rates and wages, the used capital price varies procyclically across time. The sales of capital from incumbents fluctuate procyclically in the baseline model with adverse selection, while the other two models do not replicate the procyclical sales consistent with data. However, the main business cycle statistics are indistinguishable between the baseline model and the frictionless counterpart. The long-run capital accumulation is higher in the baseline model than that in the frictionless case.

3.6.1 The Used Capital Price

3.6.1.1 The Cyclical Property of Used Capital Price

The baseline model differs from the frictionless model and fixed irreversibility model, as the relative used capital price, the measure of irreversibility, is state-dependent. Panel C of Table A.13 presents the approximate laws of motions for the baseline model. The second row in the panel displays the OLS estimations for equation (3.18). The coefficient for aggregate shock, c_k, is significantly positive, -0.03308. The number in parentheses is the standard deviation of the estimate. The coefficient for aggregate shock, c_u, is significantly negative, -0.0351. The R square is 0.9984, which indicates that the estimations approximate the true laws of motions well. Thus the two forces, aggregate shock and aggregate good capital stock, determine the evolution of used capital price over cycles.

The cyclical result is presented in Table A.17. Here I simulate the model for 5,000 periods. The time-average of used capital price is 0.9503, which is very close to the calibration result. The correlation of used capital price with output is 0.1868. Thus, used capital price is procyclical. In other words, the endogenous capital adjustment cost arising from adverse selection is countercyclical. This result supports the conjecture in Eisfeldt and Rampini (2006).

3.6.1.2 The Dynamics of Used Capital Price

Figure A.7 illustrates how the used capital price responds to a positive aggregate shock. It plots the evolution of the log-deviations of the used capital price from the steady state. Here, I simulate a model for 200 periods without aggregate shocks, then compute the used capital price with the time-averaged distribution. At period 201, the economy is hit by a positive one standard deviation shock to aggregate technology. The graph draws the log-deviation of the used capital price from period 200 to period 219, which is period 1 to period 20 in the graph. In the graph, the used capital price goes down immediately when hit by the positive shock. Then the price increases sharply in the next period. After that, the used capital price goes down with some slight bump during the process.

To understand the dynamics of the used capital price, I study the movements of different forces which shift the used capital price separately, when facing a positive aggregate shock. In the model economy, the no-arbitrage condition in equation (3.13), which reflects the interaction between new and used capital markets, determines the relative used capital price. From this equation, the consistent belief λ, the stochastic discount factor $\beta\frac{Ep'}{p}$ and the expectation of the used capital price Ep_u vary over time. Here, the stochastic discount factor, $d(A,\mu)$, is the inverse of the gross interest rate, $d(A,\mu)=\beta\frac{Ep'}{p}=\frac{1}{1+r_t(A,\mu)}$. The net interest rate is $r_t(A,\mu)$.

Figure A.9 plots the impulse response functions for interest rate and real expected used capital price $\beta\frac{Ep'p'_u}{p}$. In the bottom panel, the interest rate first goes down sharply since the positive aggregate technology shock increases the marginal product of capital. It follows a deep drop because the economy accumulates an extra amount of capital relative to the steady state. As the deviation of aggregate capital decreases, the interest rate goes back to the steady state accordingly. The stochastic discount factor moves inversely. During this process, the anticipated used capital price is above the steady state level since the level of aggregate capital is above the steady state, as the law of motions for the used capital price indicates.

The top panel in Figure A.8 plots the corresponding impulse response function for consistent belief. In equilibrium, the consistent belief equals the fraction of good capital sold in the used capital market. So, I first examine the response of the target levels for selling good capital. The right panels in Figure A.10 describe the evolutions of the target capital levels k_2^* given different idiosyncratic shocks after a positive technology shock. At period 2, when hit by the positive shock, the target capitals increase immediately, as do the threshold values. Thus, fewer firms would sell their good capital, which corresponds to the decrease of the sales of good capital as shown in the bottom panel of figure A.8. At the same time, the target capital levels for purchasing also increase, as shown in the left panel of figure A.8. The firms hold more (good) capital stock than before. In the following periods, they adjust their capital stocks gradually , and the target levels go back to the steady state values during these periods. The sales of good capital increases accordingly. It is worth noting that the sales of good capital increase dramatically at period 5, because there are larger mass of firms which adjust their capital stocks downward. As shown in figure A.10, the target levels for selling capital change slightly from period 4 to period 5. So the increase of sales mainly comes from extensive margin rather than intensive margin.

The response of used capital price combines all the above forces. With a positive aggregate technology shock, the used capital price first goes down as the interest rate goes up. To avoid the arbitrage between new and used markets, the return on used capital has to increase. The decline of consistent belief also reinforces the negative response of used capital price at period 2. In the successive period, the interest rate drops a lot, which shifts the used capital price upwards. After period 2, the interest rate increases, which decreases the used capital price accordingly. However, the quality of the used capital plays an important role in the dynamics of used capital price. At period 5, more plants sell good capital. Thus the quality of used capital increases, The higher quality induces the higher used capital price at that period, which compensates for the opposite force from the interest rate.

3.6.2 Sales of Capital

This section studies the cyclical property of sales of capital. As described before, the sales of property, plant, and equipment in Compustat data is positively correlated with output from 1971[20] to 2006. Again, sales of capital refer to the sales of used capital from all incumbents without any further notation. To compare the model results with the data facts, I simulate the economy for 5000 periods, which is about 136 simulations after dropping the initial 100 hundred periods. The same exercises are performed using the fixed irreversibility model and the frictionless model to compare the cyclical implications of sales of capital among these models. Table A summarizes the results. The corrleation in the baseline model is 0.3412, which explains about 75.2% of the correlation in the data. Both the fixed irreversibility model and the frictionless model do not replicate the procyclical sales of capital from incumbents.

In the baseline model, the sales of capital from incumbents consist of two

[20]Compustat began to collect information about SPPE in 1971.

parts: sales of good capital and sales of bad capital. As stated before, the incumbents sell all their bad capital since the average quality in the used capital market is higher than the bad capital. As for the good capital, their decisions depend on their idiosyncratic shocks, capital stocks, and aggregate states. The top panel in figure A.8 plots the responses of aggregate sales of good capital when facing a one standard deviation positive aggregate shock at period 2. The evolution of aggregate sales of capital from incumbents is in figure A.11.

Figure A.11 indicates that the aggregate sales of capital first decline at period 2, then go up at period 3. After that, the sales decrease gradually with an exception of a jump at period 5. As shown in figure A.8, the drop at period 2 is due to the increase in the sales of good capital, since the aggregate bad capital stock does not change. At period 3, the sales of good capital almost return to the steady state value. The sales of aggregate capital are above the steady state value since plants accumulate more bad capital from the higher new investment in the previous period. After that, the sales of bad capital decline along with the decrease of aggregate capital stock. At period 5, the sales of good capital increase sharply, which dominates the movement of aggregate sales of capital. From then on, sales of good and bad capital both decline and approach to the level of steady state.

Figure A.13 compares the impulse response functions of incumbents' sales of capital among the baseline model, the fixed irreversibility model, and the frictionless case. The solid line describes the evolution in the baseline model. The slashed line represents the response of sales of capital in the frictionless model(no capital adjustment cost). The dot-slashed line stands for the evolution in the fixed irreversibility model. In all these models, the sales of capital fall immediately after a positive technology shock at period 2. After that they all go up, and then decrease to the steady state level gradually.

In the frictionless model without capital adjustment cost, the target capital level only depends on the firm's idiosyncratic shock and aggregate states. In the presence of a positive aggregate shock, the target levels for firms increase since the marginal product of capital is higher than before. In the next period, the drop in the marginal product results in the decline in interest rates and target capital levels. Later on, the sales of capital go back to the steady state level as the interest rate approaches the steady state. This argument is similar to the part of the explanation from the perspective of interest rate in the baseline. In the fixed irreversibility model, the explanations for the evolution of sales of capital are the same as those in the frictionless case. The kink in the impulse response function reflects the nonlinearity of investment decisions.

Since the impulse response functions among three models have similar shapes, why do they have different cyclical implications for the sales of capital? The answer results from the magnitude. The response in the fixed irreversibility case is the most volatile. The frictionless case is the second most volatile. As shown in figure A.13, output increases at period 2, then it almost goes back the level of steady state. The small decline in the baseline model does not result in a large negative relationship with output at this period. The persistent aggregate technology shock also strengthens the positive correlation with sales of capital at a later period.

Figure A.13 compares the shifts of target levels between period 2, when facing a positive technology shock and period 1, the steady state. The two upper panels describe the shifts in the fixed irreversibility case. The two lower panels describe the movement in the baseline model. The solid line represents the targets at period 1, the steady state before the shock. The dashed line represents the targets after the shock at period 2. In the figure, the movements for two kinds of threshold values, k_1^* and k_2^*, are significant in the case with fixed irreversibility for every idiosyncratic shock level.

The shift in the target level can not explain the large differences among responses[21]. In the baseline model, the response is even smaller than that in the frictionless model. The explanation comes from the essence of the irreversibility in the baseline model, lemons. Selling bad capital is one kind of subsidy. Plants always sell all their bad capital no matter in good time or bad time. As the shift in the target level only affects the sales of good capital, the whole impact on sales of capital is small relative to other cases. The different responses of aggregate sales of capital also provide an example where the micro-level non-linearity does matter for macroeconomic analysis.

This paper aims to explain the procyclical sales of used capital after establishing the consistencies with evidences of selling price of used capital. However, other explanations may be consistent with the procyclical sales of capital. For example, the "vintage capital" implies that firms who invest in new capital sell their

[21]The difference may come from different levels of irreversibility between two models. I simulate a fixed irreversibility model where p_u^f is set to 0.95, the time-averaged used capital price in the baseline model. In this experiment, the response to shock is also large and the correlation of sales of capital is still counter cyclical.

used capital. Therefore, the more investment in a good state, the more sales of used capital in a good state. However, Eisfeldt and Rampini (2006) shows that the implication of vintage capital is not consistent with the data, since firms selling capital do not invest more than those firms who do not sell in Compustat data.[22]

3.6.3 Fraction of Used Capital Expenditure

In the model economy, the fraction of used capital expenditure over total capital expenditure is counter cyclical though both new and used capital investment[23] are procyclical. The correlation of the fraction with output is 0.7038. Data evidences also support the countercyclical fraction. Although there are only 11 observations in ACES, the corresponding correlation is significantly positive, -0.5353. The correlation is -0.5188 when using 2-digit industry level data in ACES from 1998 to 2006. Figure A.14 plots the evolution of the fraction of used capital expenditure over total capital expenditure. The fraction first drops deeply since less used capital is available in the used capital market while the demand for investment increases. In period 3, the fraction of used capital expenditure goes up, as the plants accumulate extra capital when facing the positive aggregate shock. Later, the fraction decreases but it is still above the steady state for a few periods. The fraction remains above the steady state because the higher aggregate capital stock implies

[22] As stated in their paper, "the median lagged property, plant and equipment ratio for firms selling PP&E is 21% , for those who do not sell PP&E is 23%.

[23] Here the used capital investment is the sum of sales from incumbents and sales from exiting firms. Since the exit rate is exogenous, the increased capital stock implies that the sales from exiting firms are high when the aggregate shock is high. In the model, the correlation of used capital expenditure with output is 0.4930.

higher sales of capital from exiting plants.

3.6.4 New Investment

As stated in Kahn and Thomas (2003, 2008) and Veracierto (2002), the numerical results in this paper confirm that the conventional business cycle moments are indistinguishable between the frictionless model and the model with micro-level non-linearity, i.e., the model with endogenous irreversibility. Table A.14 reports the business cycle statistics from the frictionless model and the baseline model. Most of the figures are very close between the two model results except the figure for the standard deviation of new investment with respect to output. In the frictionless case, it is 3.5274, while it is 3.181 in the model with adverse selection. The decline of volatility of investment results from the adjustment cost in the baseline model. The standard deviation of investment is 6.9238 in the frictionless model, and 6.2108 in the baseline model. The ratio is 0.8970 when the used capital price is 0.95.

3.7 Concluding Remarks

This paper studies a dynamic stochastic general equilibrium model with two separate capital markets and asymmetric information in the used capital market. The investment is partially reversible as selling good capital faces extra costs arising from information friction. The capital adjustment cost, measured as the level of irreversibility, is endogenously determined by plant investment decisions. Unlike the physical adjustment cost, the cost arising from the information problem is countercyclical. When calibrating the model with time-averaged moments, the model replicates the procyclical sales of capital from incumbents in the data. However, the model with fixed irreversibility or a frictionless model fails to explain this phenomenon. The striking difference on the moment for aggregate sales of capital provides an example that micro-level non-linearity is important in understanding the movement of macroeconomic variables.

The present paper also proposes a framework to study the impact of economic stimulus on capital reallocation, since the time-variant used capital price connects the new and used capital markets. When the government imposes investment tax credit to stimulate aggregate (new) investment during recessions, the cost of the new investment decreases. According to the no-arbitrage requirement between new and used capital markets, the used capital price must decrease. Thus, plants do not want to sell good capital as much as before. The sales and the quality of used capital decrease. As small firms invest more on used capital, this kind of policies hinders the recovery of small firms in recessions. Therefore, the policymakers should take into account the impact on capital reallocation when employing stimulating policies for new investment.

In the present paper, I assume that every firm invests the same fraction of capital on used goods. By abstracting the heterogeneity among firms, I focus on analyzing the impact of adverse selection on capital reallocation. According to ACES, small firms invest more on used capital. Eisfeldt and Rampini (2007) use micro-level evidence to show that small firms intend to invest more on used capital because of credit constraints. Thus, it would be interesting to explore the quantitative impact of credit constraints on capital reallocation in a DSGE model, where firms invest with various mixed strategies.

Appendices

Appendix A

Tables and Figures

Table A.1: Inaction Rates, Profit Shocks and GDP (1974 - 1988)

YEAR	74	75	76	77	78	79	80	81
inacta	0.07	0.08	0.08	0.07	0.07	0.07	0.07	0.07
Profshk	4.16	4.08	4.16	4.21	4.24	4.23	4.14	4.11
LGDPCA	8.37	8.37	8.42	8.47	8.52	8.55	8.55	8.57
YEAR	82	83	84	85	86	87	88	
inacta	0.09	0.10	0.08	0.07	0.10	0.08	0.10	
Profshk	4.00	4.03	4.08	4.04	4.02	4.03	4.04	
LGDPCA	8.55	8.60	8.67	8.71	8.74	8.78	8.82	

Note: The rows "inacta" represent the fraction of inactive plants each year. Plants are inactive when their (positive or negative)investment rates are less than 0.01. The rows "Profshk" stands for the average profit shocks each year. Both "inacta" and "Profshk" are calculated based on LRD database. These figures are provided by John Haltiwanger. Please refer to Cooper and Haltiwanger (2006) on the details of the estimation of profit shocks and the calculation of investment rates. The rows "LGDPCA" are the logarithm of real GDP per year. These data is from FRED database.

Table A.2: Adjustment cost estimates

Parameter	Estimate	
ν	0.0308	
Coefficient of convex adjustment cost	(0.002)	
α_1	32.06	
Coefficient of Irreversibility	(1.915)	
α_2	-36.07	
Coefficient of Cyclical Irreversibility	(2.116)	
λ	0.8370	
Coefficient of non-convex adjustment cost	(0.006)	
Moments	Data	Simulated
Serial correlation of Investment Rates	0.058	0.051
Correlation of Profit Shocks and Investment	0.143	0.161
Positive Investment larger than 20%	0.186	0.130
Negative Investment larger than 20%	0.018	0.050
Correlation of Average Profit Shocks and Inaction	-0.713	-0.710

Figure A.1: Fraction of Inactive Plants Over the Cycles

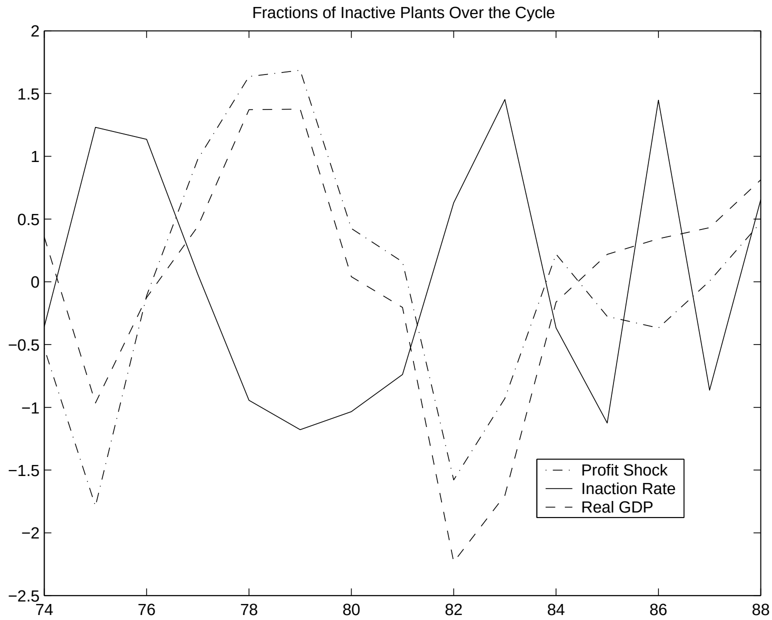


Table A.3: Parameterizations

Description	Parameters	Baseline values
Discount rate	β	0.950
Curvature of profit function	α	0.5920
Depreciation rate	δ	0.069
High level of aggregate uncertainty	A_h	1.015
Low level of aggregate uncertainty	A_l	0.985
Transition matrix of aggregate uncertainty	Π	$\begin{pmatrix} 0.750 & 0.250 \\ 0.250 & 0.750 \end{pmatrix}$
Persistence of idiosyncratic shocks	ρ_{idio}	0.885
Standard deviation of idio. innovations	σ_{idio}	0.300
Fraction of good capital in new output	q_0	0.6794
Ratio of productivity of used K to new K	ϕ	0.3958
Initial new capital stock	k_0	1.370

Table A.4: Calibration for Non-standard Parameters

Parameters	Baseline	Target Moments	Model	Data
q_0	0.6794	Fraction of used K expenditure	0.073158	0.073158
ϕ	0.3958	Entry rate	0.0144	0.0144
k_0	1.370	Fraction of inactive plants	0.0163	0.081

Table A.5: Correlation with Aggregate Uncertainty

	p_u	λ	I_u	I_n	x
Baseline	0.4988	0.4972	0.5537	0.4972	-0.5115

This table shows the correlation coefficients between equilibrium outcomes and aggregate uncertainty which includes used capital price p_u, consistent belief λ, aggregate used and capital expenditure I_u and I_n. These results based on the 10000-period simulation of the baseline economy. The first 100 periods are dropped.

Table A.6: Sensitivity Analysis: Correlation with Aggregate Uncertainty

$\pi_{hh} = \pi_{ll}$	p_u	λ	I_u	I_n
0.65	0.2969	0.2950	0.6319	0.2950
0.70	0.3902	0.3884	0.5613	0.3884
0.75	0.4990	0.4972	0.5537	0.4972
0.77	0.5379	0.5364	0.5373	0.5364

The table presents the correlations of used capital price p_u, consistent belief λ, used and new capital expenditure I_u, I_n with aggregate uncertainty when varying π_{hh} and π_{ll}. π_{hh} and π_{ll} are the probabilities to keep the current uncertainty level next period.

Table A.7: Robustness

q_0	ϕ	k_0	Corr(p_u,shock)
0.8	0.3958	1.37	0.4996
0.6794	0.4	1.37	0.4988
0.6794	0.3512	0.8	0.5005
0.8	0.4	0.8	0.4978

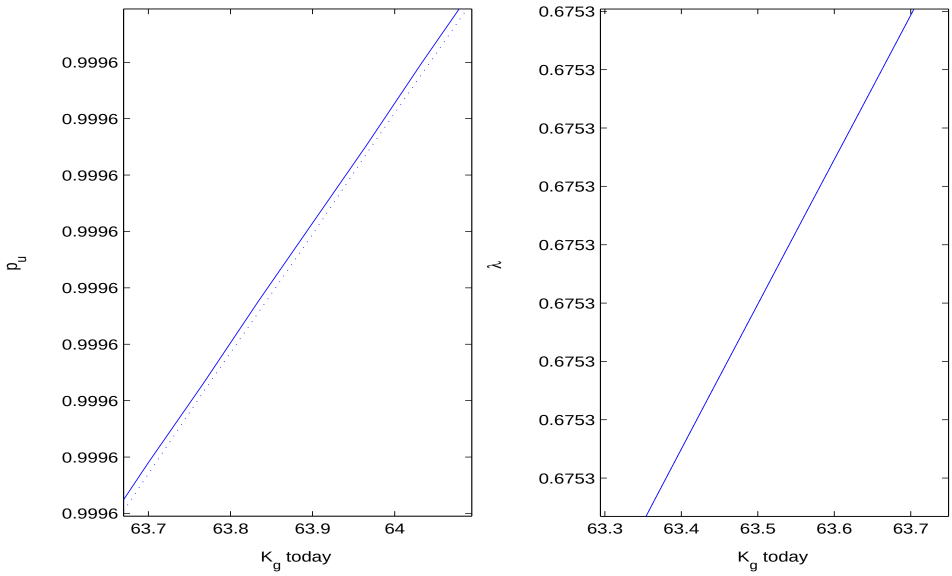
The table presents the correlation of used capital price with aggregate uncertainty with different (q_0, ϕ, k_0). The other parameterizations are the same as the baseline values.

Table A.8: The correlation of SPPE with Output

	HP-filter	Linear Trend
Correlation of SPPE with Output	0.4479	0.4537
Significance Level	(0.0055)	(0.0062)

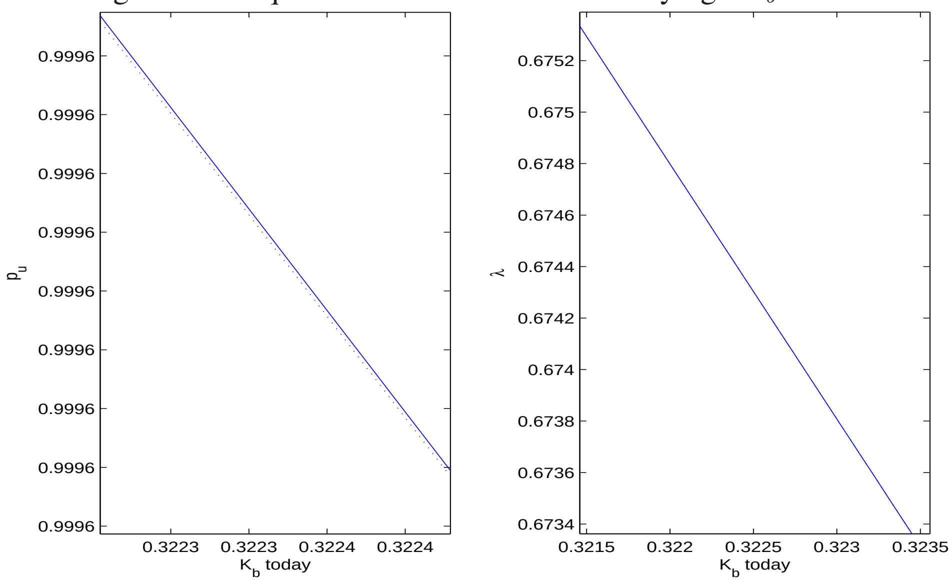
This table describes the cyclical property of sales of property,plant and equipment in Compustata , which is deflated by the CPI. The natural logarithm of the variable is used to compute the deviation from trend with HP filter or a linear trend. The significance level for each estimate is in the parentheses.

Figure A.2: Equilibrium Outcomes when varying $\bar{K}_g$ and A



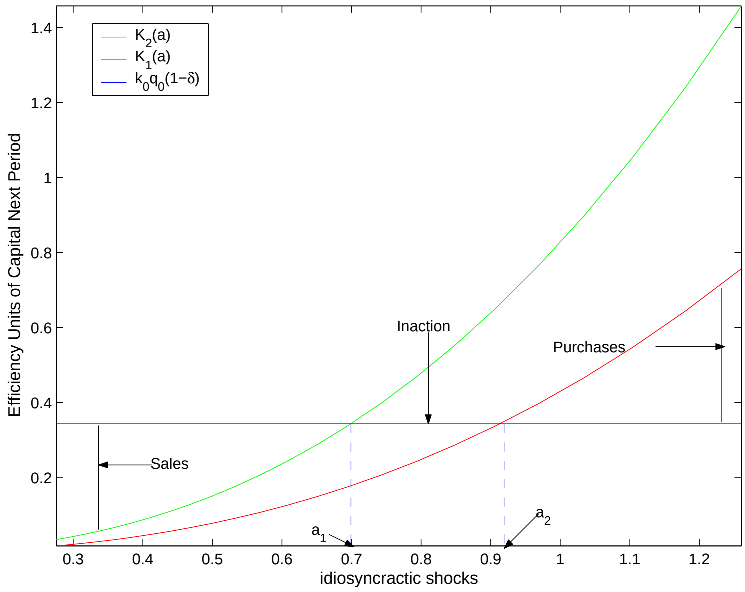
This graph plots used capital price, p_u and consistent belief, λ when varying $\bar{K}_g$ and A. The aggregate state $rK_b = 0.3215$. The dotted lines are those with low aggregate uncertainty level. Solid lines are those corresponding to high aggregate uncertainty level. The graph only shows a fraction of the function.

Figure A.3: Equilibrium Outcomes when varying rK_b and A



This graph plots used capital price, p_u and consistent belief, λ when varying rK_b and A. The aggregate state $\bar{K}_g = 62.77$. The dotted lines are those with low aggregate uncertainty level. Solid lines are those corresponding to high aggregate uncertainty level. The graph only shows a fraction of the function.

Figure A.4: Solutions to Plants' Problems



This graph describes two side (s, S) rules of plants' investment decisions in steady states. The blue line is plants' initial capital stock after selling all of their bad capital, $k_0 q_0(1-\delta)$. The green line plots the optimal choice for the efficiency units of capital $\tilde{k}'_t$ next period when selling good capital, $K_2(a)$. The red line plots the optimal choice for the efficiency units of capital next period when purchasing capital, $K_1(a)$. a_1 and a_2 are two cutoff values in Proposition 2.3.4.
Parameterizations: $\beta = 0.81707, \alpha = 0.592; \delta = 0.46138; \rho_{idio} = 0.885; \sigma_{idio} = 0.30; A_t = 1.03; q_0 = 0.70; \phi = 0.4; k_0 = 0.9158$. Discrete the idiosyncratic shocks with Tauchen's method with 40 grids. This graph shows the previous 34 grids.
Equilibrium: $p_u = 0.9616$; $\lambda = 0.6061$; $x = 0.8417$.

Table A.9: Parameterizations(Annual)

Description	Parameters	Baseline values
Discount rate	β	0.977
Growth rate	r	0.016
Depreciation rate	δ	0.088
Capital's share	α	0.2560
Labor's share	ν	0.64
Preference parameters for leisure	sl	2.4
Persistence of idio. shocks	ρ_{idio}	0.859
Standard deviation of idio. innovations	σ_{idio}	0.022
Persistence of aggregate shocks	ρ_{agg}	0.859
Standard deviation of aggregate innovations	σ_{agg}	0.014
Exit rate	η	0.01

Table A.10: Calibration for Non-standard Parameters: Baseline Model

Para.	Baseline	Target Moments	Model	Data
q_0	0.9972	Fraction of used K expenditure	0.0747	0.0750
p_c	0.6605	Discount of Selling Prices from exiter	0.6275	0.6275

Table A.11: Calibration for Non-standard Parameters: Fixed Irr. Model

Para.	Baseline	Target Moments	Model	Data
p_u	0.9824	Fraction of used capital expenditure	0.0750	0.0750
p_c	0.6387	Discount of Selling Prices from exiter	0.6275	0.6275

Table A.12: The degree of Irreversibility in models and estimation

	Baseline	Fixed Irr.	Frictionless	CH(2006)	Data
Used K price	0.9501	0.9824	1	0.981	N/A
Frac. of used K	7.474%	7.502%	28.68%	N/A	7.502%

Table A.13: Approximate Laws of Motions

	i	a_i	b_i	c_i	R^2
Panel A: Frictionless					
	k	-0.0166	0.7576	0.5259	0.9999
		(0.0000)	(0.0006)	(0.0011)	
	p	0.9046	-0.3838	-0.67806	1.0000
		(0.0000)	(0.0001)	(0.0001)	
Panel B: Fixed					
	k	-0.0063	0.7593	0.6026	1.0000
		(0.0000)	(0.0004)	(0.0007)	
	p	0.9259	-0.3807	-0.6187	1.0000
		(0.0000)	(0.0001)	(0.0003)	
Panel C: Baseline					
	k	-0.0078	0.7604	0.5162	1.0000
		(0.0000)	(0.0003)	(0.0006)	
	u	-0.0499	0.03308	-0.0351	0.9984
		(0.0000)	(0.0001)	(0.0001)	
	p	0.9244	-0.3800	-0.6854	1.0000
		(0.0000)	(0.0001)	(0.0002)	

Table A.14: Conventional RBC Moments

	Corr	w.r.t	output	STD	relative	to output
	I_n	N	C	I_n	N	C
Frictionless	0.9692	0.9437	0.9124	3.5274	0.5949	0.4808
Baseline	0.9726	0.9449	0.9183	3.181	0.5868	0.4852

Table A.15: Correlations of Sales of Capital with Output

	Endo. Irr	Frictionless	Fixed Irr.
corr(I_u, Y)	0.3412	-0.2508	-0.7517

Table A.16: Long-run Statistics

	Frictionless	Fixed Irr.	Baseline
Mean of capital	0.9192	0.9628	0.9544

Table A.17: Used Capital Price and Fraction of Used K Expenditure

	Mean of p_u	Corr(p_u,Y)	Corr(xfrac, Y)
Baseline Model	0.9503	0.1868	-0.7038

Figure A.5: Sales of Capital over the Cycle

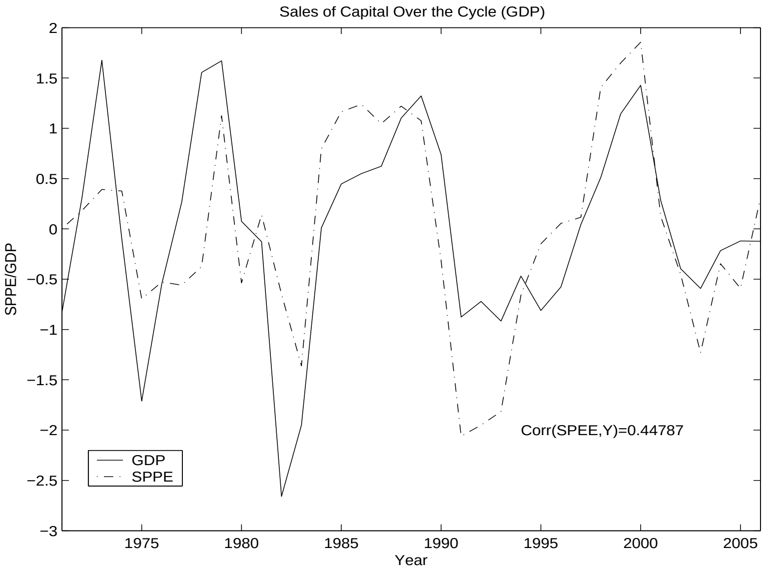


Figure A.6: Timing of the Model

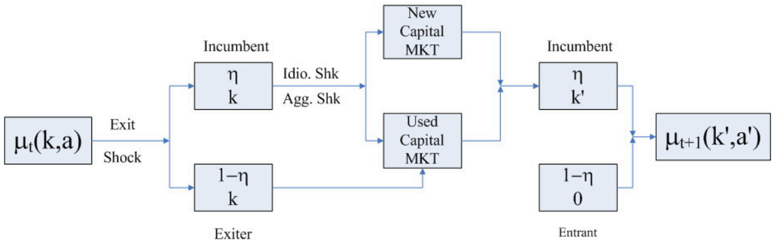
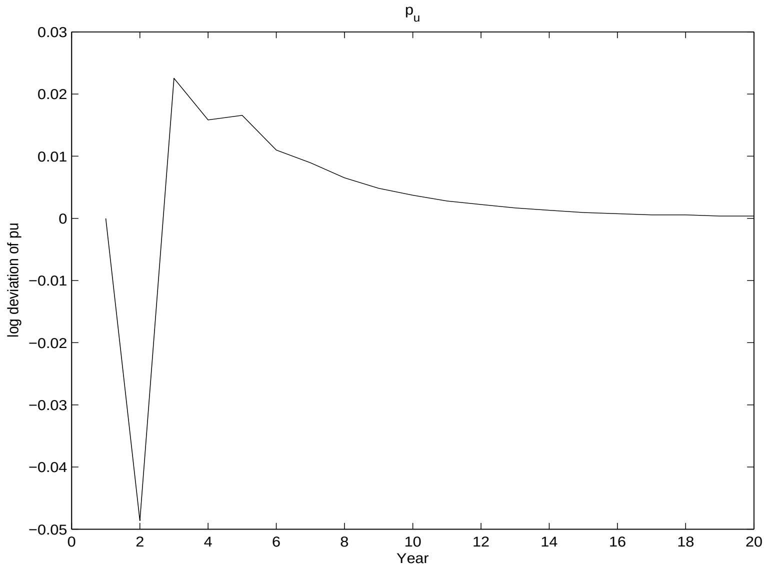
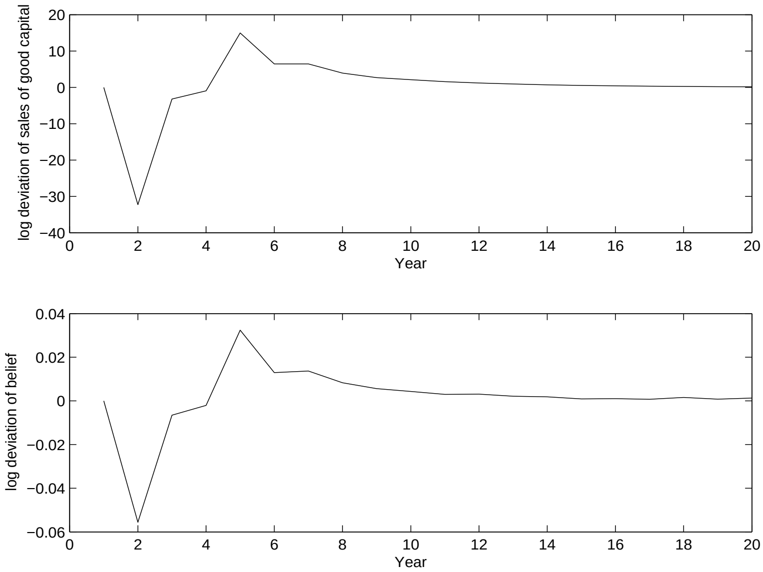


Figure A.7: Impulse Response Functions for Used Capital Price



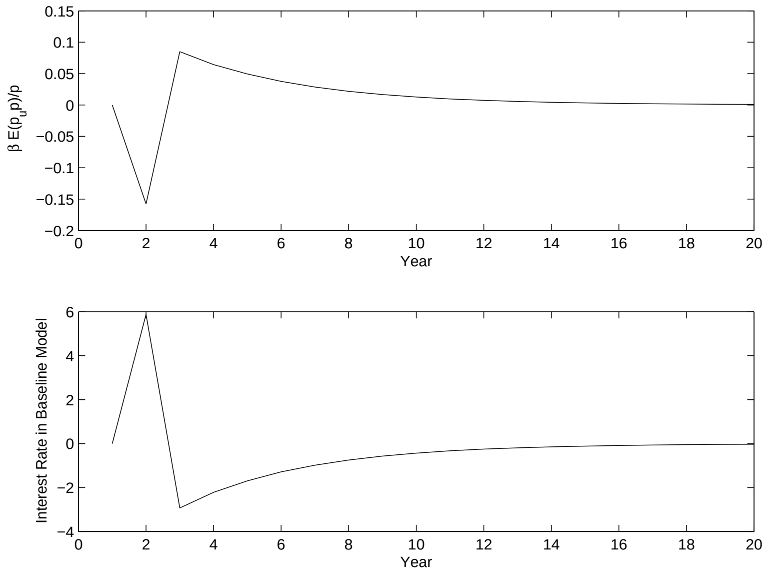
This graph plots log-deviation of used capital price from steady state values when there is one standard deviation of aggregate technology shock at period 2. Here “steady date” indicates that values comes from time-average distribution where I simulate the model for 200 periods without aggregate shocks.

Figure A.8: Impulse Response Functions for Belief and Sales of Good Capital



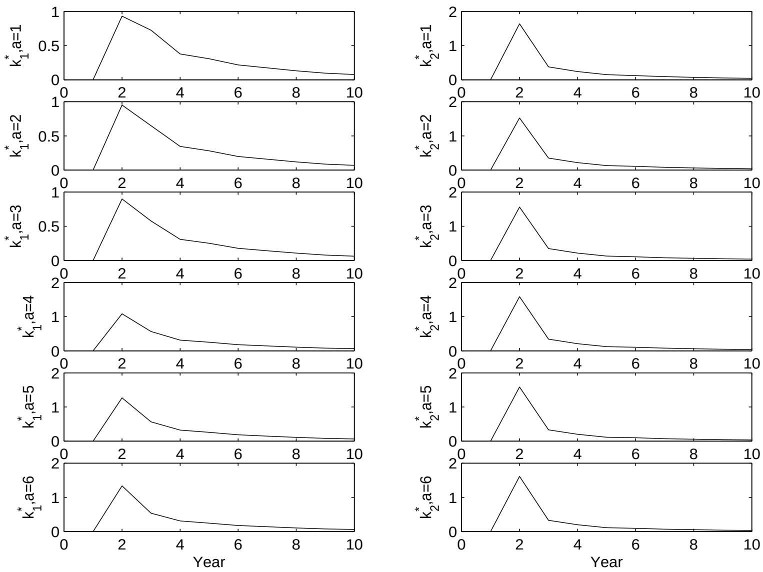
This upper panel plots log-deviation of consistent belief and the bottom panel plots sales of good capital. Here “steady date” and the shock process are the same as those in the previous graph.

Figure A.9: Impulse Response Functions for Interest Rates and Expected Price



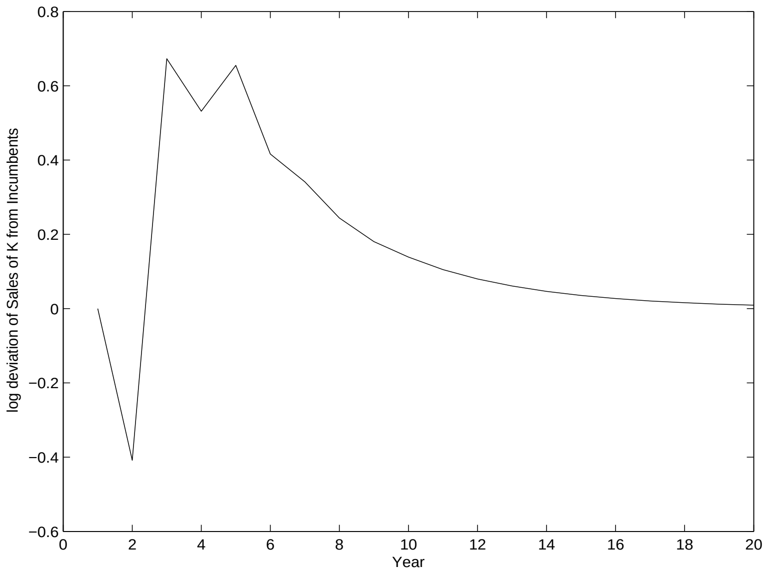
The upper panel plots log-deviation of expected real used capital price, and the bottom panel plots the log-deviation of interest rates. Here “steady date” and the shock process are the same as those in the previous graphs.

Figure A.10: Impulse Response Functions for Target Level



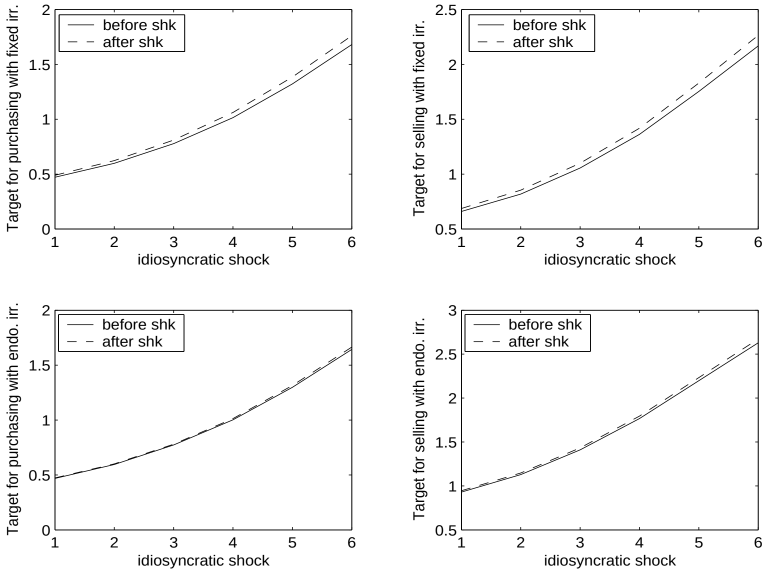
This graph plots log-deviations of two target capital level in the baseline model: k_1^* and k_2^*. The left six panels describe the responses of the low threshold values, k_1^*. The right six panels describe the responses of the upper threshold values, k_2^*. a from 1 to 6 indicates the level of idiosyncratic shocks from low to high. Here "steady date" and the shock process are the same as those in the previous graphs.

Figure A.11: Impulse Response Functions for Sales of Capital



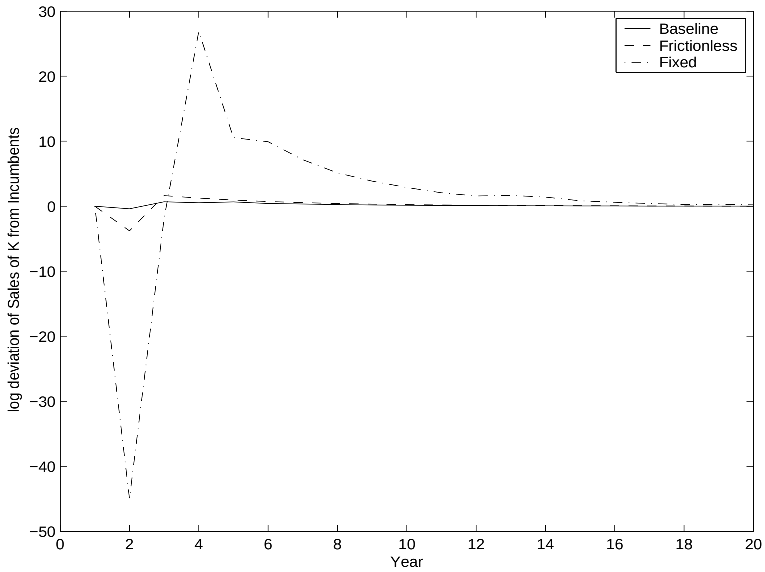
This graph plots log-deviation of sales of capital in the baseline model. Here "steady date" and the shock process are the same as those in the previous graphs.

Figure A.12: Thresholds Before and After Shocks



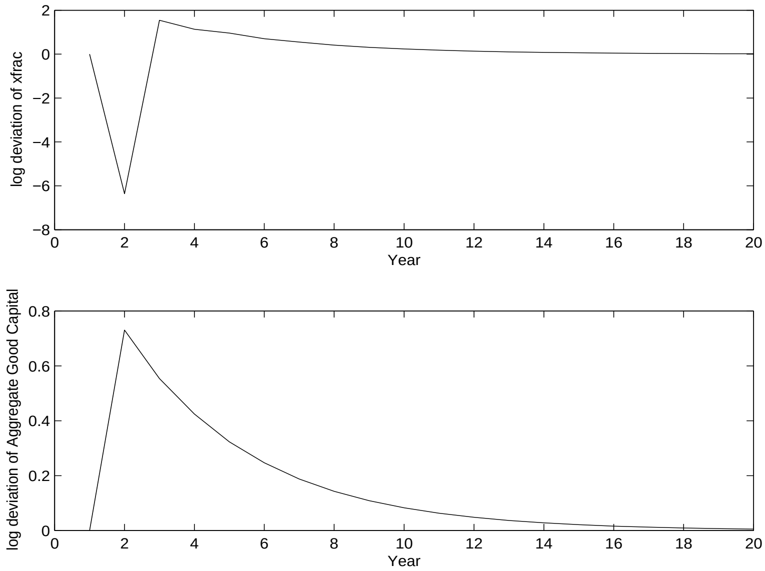
This graph plots variations of target capital levels of selling and buying capital before and after one standard deviation of aggregate technology shock. The "before shock" values come from steady state values. The "Steady State" is the same as in previous graphs. The solid lines stand for threshold values before shock. The slashed lines stand for values after shocks. The upper panels are the case with fixed irreversibility. The lower panels are the case with adverse selection.

Figure A.13: Impulse Response Functions for Sales of Capital



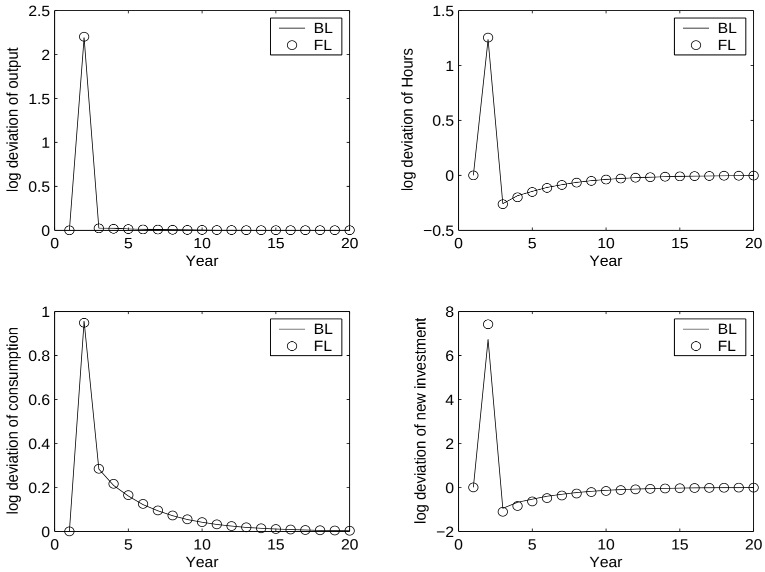
This graph plots log-deviation of sales of capital from steady state values when there is one standard deviation of aggregate technology shock at period 2. The "Steady State" is the same as in previous graphs. The solid line stand for those from baseline model. The slashed line stands for those from frictionless model. The dot-slashed line stands for those from model with fixed irreversibility.

Figure A.14: Impulse Response Functions for $xfrac$ and $\bar{K}_g$



The upper panel in this graph plots log-deviations of fraction of used capital expenditure from steady state values. The bottom panel plots the impulse response for aggregate good capital stock. The "Steady State" and shocks are the same as stated in previous graphs.

Figure A.15: Impulse Response Functions for Aggregate Variables



This graph plots log-deviation of aggregate quantities from steady state values when there is one standard deviation of aggregate technology shock at period 2. Here "steady date" indicates that values come from time-average distribution where I simulate the model for 200 periods without aggregate shocks. The solid lines stand for those from baseline model. The circles stand for those from frictionless model.

Appendix B

Appendix for Chapter 2

B.1 Proofs

Proof of Lemma 1. Denote I_g^t the aggregate sales of good capital from young plants in the used capital market at period t. I_b^t is the aggregate sales of bad capital from young plants in the used capital market at period t. I_u^t stands for the total used capital expenditure of young plants at period t. I_n^t represents the total new capital expenditure of young plants at period t. $K_g^{d,t} = k_0 q_0(1-\delta) - I_g^t$ is the aggregate good capital stock of young plants at period t.

Obviously, $\lambda_t > 0$ which comes from the assumption that old plants are forced to exit. Thus $I_b^t = k_0(1-q_0)(1-\delta)$, $I_g^t < k_0 q_0(1-\delta)$ from the following proposition 2.3.4. $\frac{I_g^t}{I_g^t+I_b^t} < q_0$.

At period 1, $\bar{K_g}^1 = \lambda_1 I_u^1 + q_0 I_n^1 + K_g^{d,1}$, $\bar{K}_b^1 + \bar{K}_g^1 = I_u^1 + I_n^1 + K_g^{d,1}$. $\frac{K_g^{d,1}+I_g^1}{K_g^{d,1}+I_g^1+I_b^1} = q_0 \Rightarrow \frac{K_g^{d,1}+I_g^1+\bar{K}_g^0(1-\delta)}{K_g^{d,1}+I_g^1+I_b^1+(1-\delta)(\bar{K}_g^0+\bar{K}_b^0)} \leq q_0$. $\because I_u^1 = I_g^1 + I_b^1 + (1-\delta)(\bar{K}_g^0 + \bar{K}_b^0)$, $\lambda_1 I_u^1 = I_g^1 + (1-\delta)\bar{K}_g^0$, $\therefore \frac{k_g^{d,1}+\lambda_1 I_u^1}{I_u^1+k_g^{d,1}} \leq q_0$. $\therefore \frac{k_g^{d,1}+\lambda_1 I_u^1+q_0 I_n^1}{I_u^1+I_n^1+k_g^{d,1}} \leq q_0$. i.e. $\frac{\bar{K}_g^1}{\bar{K}_g^1+\bar{K}_b^1} \leq q_0$.

Assume $\frac{\bar{K}_g^{t-1}}{\bar{K}_g^t+\bar{K}_b^{t-1}} \leq q_0$ at period $t-1$. Want to show $\frac{\bar{K}_g^t}{\bar{K}_g^t+\bar{K}_b^t} \leq q_0$. Similar arguments apply here. $\bar{K_g}^t = \lambda_t I_u^t + q_0 I_n^t + K_g^{d,t}$, $\bar{K}_b^t + \bar{K}_g^t = I_u^t + I_n^t + K_g^{d,t}$. $\frac{K_g^{d,t}+I_g^t}{K_g^{d,t}+I_g^t+I_b^t} = q_0 \Rightarrow \frac{K_g^{d,t}+I_g^t+\bar{K}_g^{t-1}(1-\delta)}{K_g^{d,t}+I_g^t+I_b^t+(1-\delta)(\bar{K}_g^{t-1}+\bar{K}_b^{t-1})} \leq q_0$. $\because I_u^t = I_g^t + I_b^t + (1-\delta)(\bar{K}_g^{t-1}+\bar{K}_b^{t-1})$, $\lambda_t I_u^t = I_g^t + (1-\delta)\bar{K}_g^{t-1}$, $\therefore \frac{k_g^{d,t}+\lambda_t I_u^t}{I_u^t+k_g^{d,t}} \leq q_0$. $\therefore \frac{k_g^{d,t}+\lambda_t I_u^t+q_0 I_n^t}{I_u^t+I_n^t+k_g^{d,t}} \leq q_0$.

i.e. $\frac{\bar{K}_g^t}{\bar{K}_g^t+\bar{K}_b^t} \le q_0$. $\square$

Proof of Proposition 2.3.1. Since $\int i_{n,t-1}(a)dF > 0$ in an equilibrium with positive aggregate investment, there always exists some fraction of good capital and some fraction of bad capital. Thus $\lambda_t \in (0,1)$ according to the condition of consistent belief.

In the used capital market at period t, the old plants sell $(1-\delta)(\bar{K}_g^{t-1}) + \bar{K}_b^{t-1}$. Young plants sell I_g^t and I_b^t, buy I_u^t. $I_u^t = I_g^t + I_b^t + (1-\delta)(\bar{K}_g^{t-1} + \bar{K}_b^{t-1})$. $\lambda_t = \frac{I_g^t+(1-\delta)\bar{K}_g^{t-1}}{I_g^t+I_b^t+(1-\delta)(\bar{K}_g^{t-1}+\bar{K}_b^{t-1})}$. $\because \frac{I_g^t}{I_g^t+I_b^t} < q_0$ and $\frac{\bar{K}_g^{t-1}}{\bar{K}_g^{t-1}+\bar{K}_b^{t-1}} \le q_0, \therefore \lambda_t < q_0$. $\square$

Proof of Proposition 2.3.2. Let the plant choose $(\tilde{k}', i_u, i_g, i_b)$, $i_n = \frac{1}{\bar{q}}(\tilde{k}' - i_u\bar{\lambda} - k_0(1-\delta)\bar{q} + i_g + i_b)$. $i_n \ge 0$. Now, the objective function of plants' problem is concave and the constraint set is convex. A local maximum is a also global maximum. Kuhn-Tucker conditions fully characterize the solutions of the maximization problem. Let $\mu_1, \mu_2, \mu_3, \mu_5$ be Lagrange Multipliers for non-negative constraints of $i_{n,t}, i_{u,t}, i_{g,t}, i_{b,t}$ respectively and μ_4, μ_6 be Lagrange Multipliers for constraints of upper bounds of $i_{g,t}, i_{b,t}$. The first order conditions are as follows:

$$\tilde{k}'_t: \quad -\frac{1}{\bar{q}} + a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1} + \mu_1\frac{1}{\bar{q}} + E\beta p_{u,t+1}(1-\delta)\frac{1}{\bar{q}} = 0 \tag{B.1}$$

$$i_{u,t}: \quad \frac{\bar{\lambda}}{\bar{q}} - p_{u,t} + E\beta p_{u,t+1}(1-\delta) - \mu_1\frac{\bar{\lambda}}{\bar{q}} + \mu_2 - E\beta p_{u,t+1}(1-\delta)\frac{\bar{\lambda}}{\bar{q}} = 0 \tag{B.2}$$

$$i_{g,t}: \quad -\frac{1}{\bar{q}} + p_{u,t} - E\beta p_{u,t+1}(1-\delta) + \frac{1}{\bar{q}}\mu_1 + \mu_3 - \mu_4 + E\beta p_{u,t+1}(1-\delta)\frac{1}{\bar{q}} = 0 \tag{B.3}$$

$$i_{b,t}: \quad -\frac{\phi}{\bar{q}} + p_{u,t} - E\beta p_{u,t+1}(1-\delta) + \mu_1\frac{\phi}{\bar{q}} + \mu_5 - \mu_6 + E\beta p_{u,t+1}(1-\delta)\frac{\phi}{\bar{q}} = 0 \tag{B.4}$$

From equation (B.1), $\mu_1\frac{1}{\bar{q}} = \frac{1}{\bar{q}} - a\beta EA_{t+1}\tilde{k}'^{\alpha-1} - E\beta p_{u,t+1}\frac{1}{\bar{q}}(1-\delta)$. Plug this into equation (B.2),(B.3) and (B.4). I have the follow equations:

$$i_{u,t}: \quad -p_{u,t} + E\beta p_{u,t+1}(1-\delta) + \bar{\lambda}EaA_{t+1}\alpha\tilde{k}'^{\alpha-1} + \mu_2 = 0 \tag{B.5}$$

$$i_{g,t}: \quad p_{u,t} - a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1} - E\beta p_{u,t+1}(1-\delta) + \mu_3 - \mu_4 = 0 \tag{B.6}$$

$$i_{b,t}: \quad p_{u,t} - a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1}\phi - E\beta p_{u,t+1}(1-\delta) + \mu_5 - \mu_6 = 0 \tag{B.7}$$

$(B.1) * \bar{\lambda}_t - (B.5) \Rightarrow -\frac{\bar{\lambda}_t}{\bar{q}} + p_{u,t} + E\beta p_{u,t+1}(1-\delta)(\frac{\bar{\lambda}_t}{\bar{q}} - 1) + \mu_1\frac{\bar{\lambda}_t}{\bar{q}} - \mu_2 = 0$. Thus an equilibrium with aggregate positive new and used capital investment must satisfy the no-arbitrage condition. □

Proof of Proposition 2.3.3. The low bound comes from no-arbitrage condition. Only need to show the upper bound. From equation (B.1), $a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1}\bar{q} > 0$, $\mu_1 \geq 0$, thus $E\beta p_{u,t+1}(1-\delta) - 1 < 0$. Rearrange the no-arbitrage condition, $p_{u,t} < 1$. □

Lemma 5. $i_{n,t}(a)i_{g,t}(a) = 0$; $i_{u,t}(a)i_{g,t}(a) = 0$; $i_{b,t} = (1-\delta)k_0(1-q_0), \forall a, t$.

Proof of Lemma 5. (B.5)+(B.7)$\Rightarrow a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1}(\bar{\lambda}_t - \phi) + \mu_2 + \mu_5 - \mu_6 = 0$. $\because \bar{\lambda}_t > \phi$, $\mu_6 > 0$. $\therefore i_{b,t} = k_0(1-\delta)(1-q_0)$.

(B.5)+(B.6)$\Rightarrow a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1}(\bar{\lambda}_t - 1) + \mu_2 + \mu_3 - \mu_4 = 0$. $\because \bar{\lambda}_t < 1$, $i_{g,t}$ and $i_{u,t}$ can't be interior solutions simultaneously. Since $i_{n,t}(a)$ and $i_{u,t}(a)$ have to be both positive or both zero, $i_{g,t}$ and $i_{n,t}$ can't be interior solutions simultaneously,too. □

Proof of Proposition 2.3.4. Since new and used capital are perfect substitutes, I assume buyers buy the same fraction of new and used capital. That is, plants choose $i_{0,t} = i_{n,t}\bar{q} + i_{u,t}\bar{\lambda}_t$ where $i_{u,t} = \frac{x_t}{\bar{\lambda}_t}$ and $i_{n,t} = \frac{1-x_t}{\bar{q}}$. $x_t \in (0,1)$ which is determined in equilibrium. Given the plant's idiosyncratic shock, I first find the cut-off value such that the plant are indifferent between purchasing capital and inaction:

$$\frac{E\beta p_{u,t+1}(1-\delta) - p_{u,t}}{\bar{\lambda}_t} + a\alpha E\beta A_{t+1}\tilde{k'}_t^{\alpha-1} = 0 \tag{B.8}$$

That is $\tilde{k'}_t = k_0 q_0(1-\delta)$.

$$\bar{a}_t = \frac{p_{u,t} - E\beta p_{u,t+1}(1-\delta)}{\bar{\lambda}_t \alpha E\beta A_{t+1}(k_0 q_0(1-\delta))^{\alpha-1}}$$

Define the cutoff value $\underline{a}_t$ such that the plant is indifferent between selling capital and inaction.

$$\underline{a}_t = \frac{p_{u,t} - E\beta p_{u,t+1}(1-\delta)}{\alpha E\beta A_{t+1}(k_0 q_0(1-\delta))^{\alpha-1}}$$

Obviously, $\underline{a}_t < \bar{a}_t$. When $a > \bar{a}_t$, the marginal benefit of investment is higher than its marginal cost, the plant then invest. When $a \leq \bar{a}_t$, the plant does not invest. When $a < \underline{a}_t$, the price-the resale value-of capital is higher than its marginal product, the plant sells its good capital. Otherwise, it doesn't adjust its capital.

□

Proof of Proposition 2.3.5. (monotonicity)Pick $f, g \in B(S)$ and $f(X) \leq g(X)$, for all $X \in S$. $\because \lambda(X) < q_0, \therefore (Tf)(X) \leq (Tg)(X)$. (discounting)$T(f+a)(X) = \frac{\bar{\lambda}(X)}{\bar{q}} + \frac{\bar{q}-\bar{\lambda}(X)}{\bar{q}}\beta(1-\delta)Ef(X') + a\frac{\bar{q}-\bar{\lambda}(X)}{\bar{q}}\beta(1-\delta)$. $\frac{\bar{q}-\bar{\lambda}(X)}{\bar{q}}\beta(1-\delta) \in (0,1)$. Thus satisfy Blackwell's sufficient conditions. □

B.2 Two idiosyncratic shock case

This section studies the steady state of a model with two idiosyncratic shocks. Suppose half of plants face high idiosyncratic shock a_h, half have low idiosyncratic shock a_l. The following proposition provides a sufficient condition for the existence of equilibrium.

Proposition B.2.1. *If there is no aggregate uncertainty and*
1) $\frac{1-\beta(1-\delta)}{\bar{q}\alpha\beta} < \frac{a_1}{(k_0 q_0(1-\delta))^{\alpha-1}} < \frac{1-\beta(1-\delta)}{\alpha\beta}$;
2) $\frac{1-\beta(1-\delta)}{\alpha\beta(\bar{q}+(\phi-\bar{q})\beta(1-\delta))} < \frac{a_h}{(k_0 q_0(1-\delta))^{\alpha-1}} < \frac{(1+\frac{1-\delta}{\delta}\frac{2((1-q_0)\phi(1+\delta)+q_0-\bar{q}\delta)}{q_0(1-\delta)})^{1-\alpha}}{\bar{q}+\beta(1-\delta)(1-\phi)q_0\delta\frac{q_0-1}{1-q_0\delta}}\frac{1-\beta(1-\delta)}{\alpha\beta}$*, then an equilibrium with aggregate positive investment exists. In particular, the young plants with high shocks purchase capital while those with low shocks only sell their bad capital.*

Proof. Suppose plants with low plants only sell bad capital and plants with high shocks purchase good capital. The used market clearing condition, consistency condition for belief and no-arbitrage condition are as follows:

$$\frac{xi_0}{\bar{\lambda}} = 2k_0(1-q_0)(1-\delta) + (\frac{x}{\bar{\lambda}} + \frac{1-x}{\bar{q}})i_0(1-\delta) + 2k_0 q_0(1-\delta)^2 \tag{B.9}$$

$$\frac{\lambda x i_0}{\bar{\lambda}} = 2k_0 q_0(1-\delta)^2 + (\frac{x\lambda}{\bar{\lambda}} + \frac{(1-x)q_0}{\bar{q}})i_0(1-\delta) \tag{B.10}$$

$$\frac{p_u}{\bar{\lambda}} = \frac{1}{(1-\beta(1-\delta))\bar{q} + \bar{\lambda}\beta(1-\delta)} \tag{B.11}$$

$$(B.9) - (B.10)/q_0 \Rightarrow \frac{\frac{\delta\lambda}{q_0} + \frac{(1-\delta)\bar{\lambda}}{\bar{q}}}{1-\frac{\lambda}{q_0}} = \frac{1-\delta}{1-q_0} + \frac{i_0}{2k_0(1-q_0)\bar{q}}.$$

Define $F(\lambda) = \frac{\frac{\delta\lambda}{q_0}+\frac{(1-\delta)\bar{\lambda}}{\bar{q}}}{1-\frac{\lambda}{q_0}} - \frac{1-\delta}{1-q_0} - \frac{i_0}{2k_0(1-q_0)\bar{q}}$. Obviously, $F(0) < 0$, $\exists\epsilon > 0$ such that $F(q_0 - \epsilon) > 0$. Thus exist an equilibrium. Now need to show the corresponding $x \in (0, 1)$. Rearrange (B.9) and (B.10),

$$\frac{\frac{\delta\lambda x}{\bar{\lambda}} - \frac{(1-\delta)(1-x)q_0}{\bar{q}}}{\frac{\delta x}{\bar{\lambda}} - \frac{(1-\delta)(1-x)}{\bar{q}}} = \frac{q_0(1-\delta)}{1-q_0+q_0(1-\delta)} < 1$$

Therefore $x > 0$. To have $x < 1$, need to show $\lambda < \lambda_0 = \frac{q_0(1-\delta)}{1-q_0\delta}$. It is sufficient to show $F(\lambda_0) > 0$. With the conditions in the proposition, only high shock plants buy capital and low shock plants only sell bad capital. And $F(\lambda_0) > 0$. $\square$

B.3 Numerical Details

The detailed computation algorithm is as follows:

1. Find an initial guess of $H_1^0(\tilde{X})$ and $H_2^0(\tilde{X})$.

2. - Guess $\lambda^0(\tilde{X})$, compute the corresponding $p_u^0(\tilde{X})$ according to (2.6).
   - Solve the young plants' problem given p_u and λ.
   - Compute $\lambda^1(\tilde{X})$ according to (2.7) and (2.8). If $\lambda^1 = \lambda^0$, stop. Otherwise update λ^0 and continue to iterate until it converges.
   - Compute the corresponding $x(\tilde{X})$.

3. Compute $H_1^0(\tilde{X})$ and $H_2^0(\tilde{X})$ from (2.9) and (2.10). Go back to step 1 and update H_1^0 and H_2^0 if they do not converge.

Appendix C

Appendix for Chapter 3

C.1 Data Appendix

Macroeconomic data: Annual real GDP is from Table GDPCA1 in Bureau of Economic Analysis. This table includes the time series of natural logarithm of real gross GDP in billions of 2000 dollars from 1971 to 2006. Annual CPI data is for all urban consumers (current series) from the Bureau of Labor Statistics.

Sales of Property, Plant and Equipment: Data on Sales of Property, Plant and Equipment is from Compustat North America in Wharton Research Data Services (WRDS) between 1971 and 2006. The data item is SPPE in Fundamentals Annual in its website (http://wrds.wharton.upenn.edu/ds/comp/index.shtml). In Compustat, the data reporting period is fiscal year. If a company's fiscal-year-end is June 30, the annual data in 2004 represents the sales for six months of 2003 and six months of 2004. To explore the cyclical property more precisely, I assign the sales of each company into two years, which represent the fraction of sales happening in the calendar year, among the sales of the total fiscal year, if the end of fiscal year is not the same as the end of calendar year. In the above example, the sales in 2004 for the company incude half from the report in 2004 and half from the report in 2005 when I adjust the data. I exclude the observations with a combined figure code and observations outside America.

Fraction of Used Capital Expenditure: The mean fraction is calculated with data in Annual Capital Expenditure Survey(ACES) between 1995 and 2005. The formula for computing the fraction, x_{frac}, is

$$x_{frac} = \frac{U_{equ} + U_{strc}}{U_{equ} + U_{strc} + N_{equ} + N_{strc}} \tag{C.1}$$

where U_{equ} is used equipment expenditure, U_{strc} is used structure expenditure, N_{equ} is new equipment expenditure and N_{strc} is new structure expenditure. These data come from tables in annual reports. In particular, they are table 1 in 1995 report, table 1.b in 1997, table 1.b in 1998, table 1.c in 1999 and table 1.b in 2000-2006.

Columbia Industrial Survey: The sales of fixed assets used in the paper are the sum of values for sales of fixed assets excluding lands: building and structures (I24), machinery and equipment(I25), transportation equipment (I26), office equipment (I27). The data item for total employment is L1.

C.2 Proofs

Assumption 2. $p(A, \mu)$, $p_u(A, \mu)$ are bounded continuous functions. $(A, \mu) \in Z_A \times \mathcal{D}$.

As the production function is unbounded, I restrict the state space first. The first step is to construct a "help" model whose feasible set contains that in the problem (3.2). I find the boundary of the "help" model and apply the feasible correspondence for the rest of the paper.

In the “help” model, I model the exit shock as a zero technology shock with $Pr(a_i > 0|a = 0) = 0$ and $Pr(a_i = 0|a = 0) = 1$. Denote $Z_a^0 = \{0, a_1, ...a_{n_a}\}$. $\tilde{S} = E_g \times E_b \times Z_a^0$. The new transition function is Q^0 which is monotone and has feller property by assumption. The difference from the benchmark model is that the plant with zero productivity shock can choose when to exit the market and liquidate its capital. Thus to exit immediately in the benchmark model is one choice in the “help” model.

$$
\begin{aligned}
F = {} & f(k_g, a, A) - \frac{rk'_g}{q_0 - \lambda}(1 - \lambda - p_u(1 - q_0)) + \frac{rk'_b}{q_0 - \lambda}(\lambda - p_u q_0) \\
& + \frac{i_b}{q_0 - \lambda}(\lambda(1 - p_u)) + \frac{k_g(1 - \delta)}{q_0 - \lambda}(1 - \lambda - p_u(1 - q_0)) \\
& - \frac{k_b(1 - \delta)}{q_0 - \lambda}(\lambda - p_u q_0) - \frac{i_g}{q_0 - \lambda}(1 - \lambda - p_u(1 - \lambda))
\end{aligned}
$$

The rest of the “help” model is the same as the benchmark model.

The upper bound of capital stock is the level next period which can’t reimburse the output and capital stock in this period. Let a_m and A_m be the maximums in Z_a^0 and Z_A. Let p_u^m be the upper bound of $p_u(\mu, A)$. I define k_g^m such that $f(k_g^m, a_m, A_m) - (rk_g^m - k_g^m(1 - \delta))\frac{1}{q_0 - \lambda}(1 - \lambda - p_u^m(1 - q_0)) = 0$. When $\lambda < q_0$, $\frac{1}{q_0 - \lambda}(1 - \lambda - p_u^m(1 - q_0)) > 0$. Then the above term is negative if $k'_g > k_g^m$. Denote k_g^0 and k_b^0 be the maximums of initial good and bad capital stocks. The upper bound $\bar{k}_g = max(k_g^m, k_g^0)$. From equation (3.1), i_n and i_u are bounded since $i_n \geq 0$, $i_u \geq 0$. The bounds are I_n and I_u. From equation (3.1), k'_b is also bounded given k_b^0 as i_n and i_u are bounded. Denote the upper bound for k'_b as $\bar{k}_b$.

Proof of Proposition 3.4.1. Let $C(X)$ be a space of bounded and continuous functions. For $v \in C(X)$, I define the following operator,

$$Tv(k_g, k_b, a; A, \mu) = max_{k'_g, k'_b, i_g, i_b} F(k_g, k_b, z)p + \beta\eta \int v(k'_g, k'_b, z', \mu')Q(z, dz')$$
$$+ \beta(1-\eta)p_c \int (k'_g + k'_b)(1-\delta)p'_u p' Q(z, dz') \tag{C.2}$$

Tv is bounded. I apply the Maximum Theorem to show the continuity of Tv.

The first step is show that Γ is non-empty, compact-valued, continuous. Pick $x = (s, A, \mu) \in X$. Γ is non-empty because $(0, 0, k_g(1-\delta), k_b(1-\delta)) \in \Gamma(x)$. Now to prove $\Gamma(x)$ is compact-valued. Since Y is compact, $\Gamma(x) \subset Y$, need to prove $\Gamma(x)$ is closed. Given x, $i_n(x)$ is continuous. Define $\Gamma_1(x) = \{y \in Y : i_n \geq 0\}$. $\forall \{y_n\} \subset \Gamma_1(x)$, with $\{y_n\} \to y_0$, $i_n(y_n) \geq 0$, thus $i_n(y_0) \geq 0$. Then $\Gamma_1(x)$ is closed. Similarly define $\Gamma_2(x) = \{y \in Y : i_u \geq 0\}$. It is also closed. $\Gamma(x) = \Gamma_1(x) \cap \Gamma_2(x)$ is closed.

To prove Γ is continuous, I need to show it is both l.h.c and u.h.c. For l.h.c, I need to show $\forall y \in \Gamma(x)$, $\forall x_n \to x$, $\exists y_n \to y$ and $y_n \in \Gamma(x_n)$. Pick $x = (k_g, k_b, a, A, \mu)$, $y = (k'_g, k'_b, i_g, i_b) \in \Gamma(x)$, $x_n = (k_{gn}, k_{bn}, a^n, A^n, \mu^n)$. Then $i_n = \frac{1}{q_0 - \lambda}(rk'_g(1-\lambda) - rk'_b\lambda - (k_g(1-\lambda) - k_b\lambda)(1-\delta) + i_g(1-\lambda) - i_b\lambda)$, $i_u = \frac{1}{\lambda - q_0}(rk'_b(1-q_0) - rk'_b q_0 - (k_g(1-q_0) - k_b q_0)(1-\delta) + i_g(1-q_0) - i_b(1-q_0))$If $k_g > 0$ and $k_b > 0$, let $\nu_1 = \frac{i_g}{k_g(1-\delta)}$, $\nu_2 = \frac{i_b}{k_b(1-\delta)}$. $y_{3n} = i_{gn} = \nu_1 k_{gn}(1-\delta)$. $y_{4n} = i_{bn} = \nu_2 k_{bn}(1-\delta)$. Thus $lim y_{3n} \to y_3$, $lim y_{4n} \to y_4$. Let $y_{1n} = k'_{gn} = k_{gn}(1-\delta) - i_{gn} + (i_n q_0 + i_u\lambda)$, $y_{2n} = k'_{bn} = k_{bn}(1-\delta) - i_{bn} + i_n(1-q_0) + i_u(1-\lambda)$. Therefore $i_{nn} = i_n, i_{un} = i_u$. $lim k'_{gn} = k'_g$, $lim k'_{bn} = k'_b$. When $k_g = 0$, let $i_{gn} = 0$. The rest of argument is the same.

For u.h.c., need to show $\forall x_n \to x$, $\forall \{y_n\} \in \Gamma(x_n)$, $\exists$ a subsequence $\{y_{nk}\}$ such that $\lim_{n_k \to 0} y_{nk} \in \Gamma(x)$. Since Y is compact, $\{y_n\} \in Y$ always has a convergent subsequence y_{nk}. Now need to show $y = lim y_{nk} \in \Gamma(x)$. $y_{nk} \in \Gamma(x_{nk})$, then $y_{3,nk} \in [0, x_{1,nk}(1-\delta)]$,$y_{4,nk} \in [0, x_{2,nk}(1-\delta)]$,$i_{n,nk} \geq 0$ and $i_{u,nk} \geq 0$. Since i_n and i_u are continuous, $y \in \Gamma(x)$. Since x is arbitrarily picked, Γ is non-empty,compact valued, continuous.

The first term on the right-hand side of (C.2) is continuous as F, p and p_u are continuous. Now to prove the the continuity of the second term. When n is large enough,

$$\begin{aligned}
&\int v(k_g'^n, k_b'^n, a'^n, A'^n, \mu'^n)Q(a^n, A^n, da', dA') \\
&= \int v(k_g'^n, k_b'^n, a', A'^n, \mu'^n)Q(a, A^n, da', dA')
\end{aligned}$$

Next I show the next term converges to zero.

$$\begin{aligned}
&\mid \int_Z v(k_g'^n, k_b'^n, a', A'^n, \mu'^n)Q(a, A^n, da', dA') - \int_Z v(k_g', k_b', a', A', \mu')Q(a, A, da', dA') \mid \\
&\leq \mid \int_Z v(k_g'^n, k_b'^n, a', A'^n, \mu'^n)Q(a, A^n, da', dA') \\
&- \int_Z v(k_g', k_b', a', A', \mu')Q(a, A^n, da', dA')| \\
&+ | \int_Z v(k_g', k_b', a', A', \mu')Q(a, A^n, da', dA') - \int_Z v(k_g', k_b', a', A', \mu')Q(a, A, da', dA')|
\end{aligned}$$

The second term vanishes because Q has Feller property. The first term also disappears. Since $(k_g'^n, k_b'^n, a',$
$A'^n, \mu'^n) \to (k_g', k_b', a', A', \mu')$, there is a compact set $D \subset X$ such that $(k_g'^n, k_b'^n, a',$
$A'^n, \mu'^n) \in D$ and $(k_g', k_b', a', A', \mu') \in D$ for all n large enough. Since v is continuous, it is uniform continuous in D. Thus $\forall\ \epsilon > 0$, there exit $N > 1$ such that for all

$n > N$, $|v(k_g'^n, k_b'^n, a', A'^n, \mu'^n) - v(k_g', k_b', a', A', \mu')| < \epsilon$. Thus the first term vanishes too. Thus the second term in the right-hand side of (C.2) is continuous. The argument for the continuity of last term in (C.2) is the same. Thus Tv is continuous by the Maximum Theorem. $T : C(X) \to C(X)$.

T satisfies the Blackwell sufficient conditions thus it is a contraction mapping. After applying the contraction mapping theorem, the problem (3.2) has a unique fixed point $v \in C(X)$.

$\square$

Now I change the state and control variables as follows:$k = k_g + k_b$, state variables are $y_2(k, k_g, a, A, \mu)$
$\in X_2 = [0, \bar{K}] \times [0, \bar{k}_g] \times Z \times \mathfrak{D}$, control variables are $(k', k_g', i_n, i_u) \in Y_2$. $Y_2 = [0, \bar{K}] \times [0, \bar{k}_g] \times [0, I_n] \times [0, I_u]$, $\Gamma(y_2) = \{y \in Y_2 : k_g(1-\delta) + i_n q_0 + i_u \lambda - r k_g' \in [0, k_g(1-\delta)]; (k-k_g)(1-\delta) + i_n(1-q_0) + i_u(1-\lambda) - r(k'-k_g') \in [0, (k-k_g)(1-\delta)]\}$.

The recursive problem (FE2) is as follows:

$$v(k, k_g, a; A, \mu) = max_{y \in \Gamma(x_2)}(f(k_g, a, A) + (p_u - 1)i_n + p_u k(1-\delta) - p_u r k')p$$
$$+ \beta(1-\eta)(1-\delta)k' E(p_u' p')p_c + \beta\eta \int v(k', k_g', a'; A', \mu')Q(z, dz')$$

Proof of Proposition 3.4.2. Under my assumption, the integration preserves the boundedness, monotonicity and concavity. Obviously, the FE2 has unique solution $v \in C(X_2)$. Now need to prove v is strictly increasing in the second element k_g. I will use the Corollary 1 of the contraction mapping theorem in Stockey and Lucas(1989)(SL). Let $v \in C'(X_2)$ where $C'(X_2)$ is the space of continuous and (weakly)increasing functions. Then,

$Tv(k, k_g, a; A, \mu) = \max_{(k', k'_g, i_n, i_u, n)} \{(aAf(k_g) + (p_u - 1)i_n + p_u k(1-\delta) - p_u r k')p$

$+ \beta(1-\eta)(1-\delta)k' E(p'_u p')p_c + \beta\eta \int v(k', k'_g, a'; A', \mu')Q(z, dz')\}$

$= (f(k_g, a, A) + (p_u - 1)\tilde{i}_n + p_u k(1-\delta) - p_u r\tilde{k}')p + \beta(1-\eta)(1-\delta)\tilde{k}' E(p'_u p')$

$+ \beta\eta \int v(\tilde{k}', \tilde{k}'_g, a'; A', \mu')Q(z, dz')$

where $(\tilde{k}', \tilde{k}'_g, \tilde{i}_n, \tilde{i}_u) \in \underset{(k', k'_g, i_n, i_u, n)}{argmax} (f(k_g, a, A) + (p_u - 1)i_n + p_u k(1-\delta) - p_u r k')p +$ $\beta(1-\eta)(1-\delta)\tilde{k}' E(p'_u p')p_c + \beta\eta \int v(k', k'_g, a'; A', \mu')Q(z, dz')$.

Take $\hat{k}_g > k_g$, then,

$Tv(k, k_g, a; A, \mu) < (aAf(\hat{k}_g) + (p_u - 1)\tilde{i}_n + p_u k(1-\delta) - p_u r\tilde{k}')p$

$+ \beta(1-\eta)(1-\delta)\tilde{k}' E(p'_u p')p_c +$

$\beta\eta \int v(\tilde{k}', (\tilde{k}'_g + (1-\delta)(\hat{k}_g - k_g)), a'; A', \mu')Q(z, dz')$

$\leq \max_{(\hat{k}', \hat{k}'_g, \hat{i}_n, \hat{i}_u)} \{(aAf(\hat{k}_g) + (p_u - 1)\hat{i}_n + p_u k(1-\delta) - p_u r\hat{k}')p$

$+ \beta(1-\eta)(1-\delta)\hat{k}' E(p'_u p') + \beta\eta \int v(\hat{k}', \hat{k}'_g, a'; A', \mu')Q(z, dz')\}$

$= Tv(k, \hat{k}_g, a; A, \mu)$

Note $(k', \hat{k}'_g + (1-\delta)(\hat{k}_g - k_g), i_n, i_u) \in \Gamma(k, \hat{k}_g, a; A, \mu)$ since i_g is the same as before. Hence $T : C'(X_2) \to C''(X_2)$ where $C''(X_2)$ is the space of continuous and increasing function, strictly increasing in the second element. Therefore $v \in C''(X_2)$.

Suppose $y^* = (k', k'_g, i_n, i_u) \in \Gamma(x_2)$ is the optimal solution. Define $i_g = k_g(1-\delta) + i_n q_0 + i_u\lambda - rk'_g$, $i_b = (k - k_g)(1-\delta) + i_n(1-q_0) + i_u(1-\lambda) - r(k' - k'_g)$; Then $i_g \in [0, k_g(1-\delta)]$ and $i_b \in [0, (k - k_g)(1-\delta)]$.

If $i_b < (k - k_g)(1-\delta)$, pick an arbitrarily small ϵ such that $i_b + \epsilon < (k - k_g)(1-\delta)$. Let the plant sell ϵ unit of bad capital and buy the same amount of used capital. Then $(k', k'_g + \epsilon\lambda, i_n, I_u + \epsilon) \in \Gamma(x_2)$, the correspond i_n is unchanged. But $v(k', k'_g + \epsilon\lambda, a'; A', \mu') > v(k', k'_g, a'; A', \mu')$ if $\lambda > 0$. As the integration preserves monotonicity[1]. $\int v(k', k'_g + \epsilon\lambda, z', \mu')Q(z, dz') > \int v(k', k'_g, z', \mu')Q(z, dz')$.Thus y^* is not optimal. Contradiction.

If $i_g > 0$ and $i_u > 0$, pick an arbitrarily small ϵ such that $i_g - \epsilon \geq 0$ and $i_u - \epsilon \geq 0$. Consider the act: decrease the sales of good capital by ϵ unit and decrease the purchase of used capital by the same amount. Then $(k', k'_g + \epsilon(1-\lambda), i_n, i_u - \epsilon) \in \Gamma(x_2)$, i_b is unchanged. Since $v(k', k'_g + \epsilon(1-\lambda), z', \mu') > v(k', k'_g, z', \mu')$, $\int v(k', k'_g + \epsilon(1-\lambda), z', \mu')Q(z, dz') > \int v(k', k'_g, z', \mu')Q(z, dz')$. y^* is not optimal. Contradiction . $\square$

Since plants sell all their bad capital, I plug $i_b = k_b(1-\delta)$ in to equation (3.2) and have a new one period return function $F_1(k_g, k_g, k'_g, k'_b, i_g, a; A, \mu)$:

$$\begin{aligned} F_1 = (f(k_g, a, A) - \frac{rk'_g}{q_0 - \lambda}(1 - \lambda - p_u(1 - q_0)) + \frac{rk'_b}{q_0 - \lambda}(\lambda - p_u q_0))p \\ + \frac{k_g(1-\delta)}{q_0 - \lambda}(1 - \lambda - p_u(1 - q_0))p - \frac{i_g}{q_0 - \lambda}(1 - \lambda - p_u(1 - \lambda))p \\ + k_b(1-\delta)p_u + \beta(1-\eta)(k'_g + k'_b)(1-\delta)E(p'_u p')p_c \end{aligned}$$

Proof of Lemma 3. Pick an arbitrary $(a, A, \mu) \in Z \times \mathcal{D}$. Denote $x = (k_g, k_b)$, $y = (k'_g, k'_b, i_g)$. Let $v^n(x; a, A, \mu)$ be the nth-iteration value function. For convenient,

[1]The proof is similar to the lemma 9.5 in SL.

Define the following operator,

$$v^{n+1}(x; a, A, \mu) = Tv^n(x; a, A, \mu) = \underset{y \in \Gamma(x;a,A,\mu)}{max} F_1 + \beta\eta \int v^n(x'; a', A', \mu')Q(z, dz')$$

Need to show $T : C^1(X) \to C^1(X)$. $C^1(X)$ is the space for continuous functions and partially differentiable with respect to the 2nd element of x, or k_b. Obvious, the solution of problem is associated with a saddle point for the Lagrangian function.

$$L^n(x, y, a, A, \mu, m) = F_1 + \beta\eta \int v^n(x'; a, A, \mu, m)Q(z, dz') + \sum_{i=1}^{i=2} m_i g_i(x; a, A, \mu)$$

where $m = (m_1, m_2)$. m_1 is the Lagrangian multiplier for $i_n \geq 0$, $g_1 = i_n$. μ_2 is the Lagrangian multiplier for $i_u \geq 0$, $g_2 = i_u$. The constrained maximization problem can be represented as a saddle-point problem for the Lagrangian function. That's because F, v^n are concave, $E_g \times E_b \times [0, \bar{k}_g(1 - \delta)]$ and the constraints are convex and I can easily find a $\hat{y}$ such that $i_n > 0$,$i_u > 0$. The value of the constrained maximization problem equals the saddle value of the Lagrangian function.

$$Tv(x; a, A, \mu) = min_{m \geq 0} max_y \ \ L^n(x, y, a, A, \mu, m)$$

Next I show that L^n is partially differentiable with respect to k_b. Pick $x^* \in int(E_g \times E_b)$. Pick a x' in the neighborhood of such that $x^* = x'$ except that $|x_2^* - x_2'| < \epsilon$. The second element of x is k_b. Assume $x_2^* > x_2'$ without loss of generality. Let (x^*, y^*, a, A, μ, m) be a saddle point of $L^n(x^*, y^*, a, A, \mu, m)$, (x', y', a, A, μ, m) be a saddle point of $L^n(x', y, a, A, \mu, m)$. In the following argument, the aggregate states and the idiosyncratic shock are omitted. By the definition of saddle point,

$$\forall y \in \Gamma(x^*), \mu \in \mathbb{R}_+^4, L^n(x^*, y, m^*) \leq L^n(x^*, y^*, m^*) \leq L^n(x^*, y^*, m)$$

$$\forall y \in \Gamma(x'), \mu \in \mathbb{R}^4_+, L^n(x', y, m') \le L^n(x', y', m') \le L^n(x', y', m)$$

Then,

$$L^n(x^*, y', m^*) \le L^n(x^*, y^*, m^*) \le L^n(x^*, y^*, m')$$

$$L^n(x', y^*, m') \le L^n(x', y', m') \le L^n(x', y', m^*)$$

Then,

$$\begin{aligned} L^n(x^*, y', m^*) - L^n(x', y', m^*) &\le L^n(x^*, y^*, m^*) - L^n(x', y', m') \\ &\le L^n(x^*, y^*, m') - L^n(x', y^*, m') \end{aligned}$$

As k_b are not in the constraint $i_n \ge 0$ or $i_u \ge 0$ ($i_b = k_b(1-\delta)$),

$$(1-\delta)p_u p \le \frac{L^n(x^*, y^*, m^*) - L^n(x', y', m')}{x^* - x'} \le (1-\delta)p_u p$$

Therefore $L^n_2(x^*, y^*, m^*) = \underset{|x^*-x'|\to 0}{lim} \frac{L^n(x^*,y^*,m^*)-L^n(x',y',m')}{x^*-x'} = (1-\delta)p_u$. $\{L^n_2\}$ converges uniformly to $(1-\delta)p_u p$. Here $L^n(x^*, y^*, m^*) = v^{n+1}(x^*)$. Thus $v^n_2(x;)$ converges uniformly to $(1-\delta)p_u p$. According to Theorem 9.13 in Apostol(1974), v is partially differentiable with respect to k_b. $v_2(x;) = (1-\delta)p_u p$. $\square$

Proof of Proposition 3.4.4. The Kuhn-Tucker condition for the problem with respect to k_b is:

$$\begin{aligned} &\frac{\lambda - p_u q_0}{q_0 - \lambda} + \beta(1-\eta)(1-\delta)E(p'_u p')p_c + \beta\eta \int \partial v_2(k'_g, k'_b, a';)Q(z, dz') \\ &+ m_1 \frac{r\lambda}{q_0 - \lambda} - m_2 \frac{r q_0}{q_0 - \lambda} = 0 \end{aligned}$$

∂v_2 stands for the subdifferential at that point. Since v is partially differentiable with respect to k_b, $\partial v_2 = p_u p(1-\delta)$. If $\frac{\lambda - p_u q_0}{q_0 - \lambda} + \beta(1-\eta+\eta p_c)(1-\delta)E(p'_u p') > 0$, $m_2 > 0$. $i_u = 0$. Aggregate demand for used capital is zero. If $\frac{\lambda - p_u q_0}{q_0 - \lambda} + \beta(1-\eta+$

$\eta p_c)(1-\delta)E(p_u'p') < 0$, $m_1 > 0$. $i_n = 0$. Aggregate demand for new capital is zero. $\square$

Proof of Lemma 4 . The proof uses the corollary 1 of contraction mapping theorem in SL. Let S be the set of continuous functions $f(x,y) = u(x) + pp_u(1-\delta)k_b$. S is a closed set of $C(X)$. Need to prove $TS \subseteq S$. The plant has three choices on investment after selling all their capital: to sell good capital, to buy capital and inaction. The operator can be rewrite as $Tv(x) = max_{k_g}\{v^s, v^n, v^b\} + p_u p(1-\delta)k_b$. Define $u(x) = max\{v_s, v_n, v_b\}$. Want to show $u(x)$ is continuous and does not depend on k_b. As Tv is continuous, $u(x)$ is also continuous. The value function when selling good capital is:

$$v_s(k_g, a;) = max_{k_g'} f(k_g, a, A)p + p_u(k_g(1-\delta) - rk_g')p + \beta\eta \int u(k_g', a;)Q(z, dz')$$
$$+ \beta(1-\eta)(1-\delta)p_c E(p_u'p')k_g'$$

The value function of inaction is:

$$v_n(k_g, a;) = f(k_g, a, A)p + \beta\eta \int u(\frac{1}{r}k_g(1-\delta), a;))Q(z, dz')$$
$$+ \frac{1}{r}\beta(1-\eta)p_c k_g(1-\delta)^2 E(p_u'p')$$

After applying the no-arbitrage condition, the value function for buying capital is:

$$v_b(k_g, a;) = max_{k_g'} f(k_g, a, A)p + p_1(rk_g' - k_g(1-\delta))$$
$$+ \beta\eta \int u(k_g', a';)Q(z, dz') + \beta(1-\eta)(1-\delta)p_c k_g' E(p_u'p')$$

where $p_1 = \frac{1}{rq_0}(\beta(1-\delta)(\eta + (1-\eta)p_c)E(p_u'p')(1-q_0) - rp)$. $p_1 < 0$ because of $p_u \leq 1$ and no-arbitrage condition. Thus v_n, v_s and v_b do not depend on k_b. Then $Tv \subseteq S$, therefore $v = Tv \subseteq S$. $\square$

Proof of Proposition 3.4.6. First, I show that the unique fixed point of the problem 3.15 exists. The argument is similar to the proof for proposition 3.4.1. The return function in (3.14) is continuous. Let $(k_g^n, k_g^n a^n, A^n, \mu^n) \to (k_g, k_g', a, A, \mu)$. When $rk_g' = k_g(1-\delta)$, $F = f(k_g, a, A)p + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p')$. No matter $rk_g^n > k_g^n(1-\delta)$ or $rk_g^n < k_g^n(1-\delta)$, the limit is the same. Thus F is continuous. Γ is fixed. After applying the similar argument for the continuity under integration, the unique fixed point can be obtained after using Blackwell conditions and Contraction Mapping Theorem.

Next, I show that v is strictly concave in k_g. F is strictly concave in k_g. That is, for any $k_g \in E_g$,$\tilde{k}_g \in E_g$, for any $\theta \in (0,1)$, $k_g^\theta = \theta k_g + (1-\theta)\tilde{k}_g$. Need to show $F(k_g^\theta, k_g'^\theta, a) \geq \theta F(k_g, k_g', a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}_g', a;)$, the inequality is strict when $k_g \neq k_g'$. We have eight cases:

$$Case\ \ 1:\ \ rk_g' > k_g(1-\delta);\ \ r\tilde{k}_g' > \tilde{k}_g(1-\delta);\ \ rk_g'^\theta > k_g^\theta(1-\delta)$$

$$Case\ \ 2:\ \ rk_g' < k_g(1-\delta);\ \ r\tilde{k}_g' < \tilde{k}_g(1-\delta);\ \ rk_g'^\theta < k_g^\theta(1-\delta)$$

$$Case\ \ 3:\ \ rk_g' > k_g(1-\delta);\ \ r\tilde{k}_g' < \tilde{k}_g(1-\delta);\ \ rk_g'^\theta > k_g^\theta(1-\delta)$$

$$Case\ \ 4:\ \ rk_g' > k_g(1-\delta);\ \ r\tilde{k}_g' < \tilde{k}_g(1-\delta);\ \ rk_g'^\theta < k_g^\theta(1-\delta)$$

$$Case\ \ 5:\ \ rk_g' > k_g(1-\delta);\ \ r\tilde{k}_g' < \tilde{k}_g(1-\delta);\ \ rk_g'^\theta = k_g^\theta(1-\delta)$$

$$Case\ \ 6:\ \ rk_g' = k_g(1-\delta);\ \ r\tilde{k}_g' = \tilde{k}_g(1-\delta);\ \ rk_g'^\theta = k_g^\theta(1-\delta)$$

$$Case\ \ 7:\ \ rk_g' > k_g(1-\delta);\ \ r\tilde{k}_g' = \tilde{k}_g(1-\delta);\ \ rk_g'^\theta > k_g^\theta(1-\delta)$$

$$Case\ \ 8:\ \ rk_g' < k_g(1-\delta);\ \ r\tilde{k}_g' = \tilde{k}_g(1-\delta);\ \ rk_g'^\theta < k_g^\theta(1-\delta)$$

Since k_g and $\tilde{k}_g$ are symmetric, some cases are omitted. Before I prove the strict concavity of each case, I first prove $-p_1 - p_u > 0$. Let $\bar{C} = \beta(1-\delta)(\eta + (1-\eta)p_c)E(p'_u p')$. The no-arbitrage condition can be rewritten as $p_u = \frac{rp\lambda + (1_0 - \lambda)\bar{C}}{rpq_0}$. $-p_1 - p_u = \frac{1}{rq_0}(1-\lambda)(rp - \bar{C})$. Rearrange the no-arbitrage condition, $rp = \frac{q_0 - \lambda}{p_u q_0 - \lambda}\bar{C}$. Since $p_u \leq 1$ and $\lambda < 1$, $-p_1 - p_u > 0$.

Case 1, 2, and 6 are obviously because of the strict concavity of $f(k_g, a, A)$. In case 3, $F(k_g, k'_g, a;) = f(k_g, a, A)p + p_1(rk'_g - k_g(1-\delta)) + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')$, $F(\tilde{k}_g, \tilde{k}'_g, a;) = f(\tilde{k}_g, a, A)p + p_u(\tilde{k}_g(1-\delta) - r\tilde{k}'_g(1-\delta)) + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')$. $F(k^\theta_g, k'^\theta_g, a;) \geq \theta F(k_g, k'_g, a;) + (1-\theta)f(\tilde{k}_g, a, A)p - p_1(\tilde{k}_g(1-\delta) - \tilde{k}'_g) + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')) > \theta F(k_g, k'_g, a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a)$.In case 4, $F(k^\theta_g, k'^\theta_g, a;) \geq \theta(f(k_g, a, A) - p_u(rk'_g - k_g(1-\delta)) + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a;) > \theta F(k_g, k'_g, a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a;)$.In case 5, $rk'^\theta_g = k^\theta_g(1-\delta)$, then $\theta(rk'_g - k_g(1-\delta)) = (1-\theta)(\tilde{k}_g(1-\delta) - r\tilde{k}'_g)$. $F(k^\theta_g, k'^\theta_g, a;) = f(k^\theta_g, a, A) + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p') \geq \theta F(k_g, k'_g, a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a;) + (-p_1 - p_u)\theta(rk'_g - k_g(1-\delta)) > \theta F(k_g, k'_g, a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a;)$. In case 7, $F(k_g, k'_g, a;)$ is the same as before, $F(\tilde{k}_g, \tilde{k}'_g, a;) = f(\tilde{k}_g, a, A)p + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')$. $rk'^\theta_g - k^\theta_g(1-\delta) = \theta(rk'_g - k_g(1-\delta))$. $F(k^\theta_g, k'^\theta_g, a;) = (f(k^\theta_g, a, A) - \theta f(k_g, a, A) - (1-\theta)f(\tilde{k}_g, a, A)p + \theta F(k_g, k'_g, a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a)$. In case 8, $F(k_g, k'_g, a;) = f(k_g, a, A) + p_u(rk'_g - k_g(1-\delta)) + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')$, $F(\tilde{k}_g, \tilde{k}'_g, a) = f(\tilde{k}_g, a, A)p + \beta(1-\delta)(1-\eta)p_c k'_g E(p'_u p')$. $k'^\theta_g - k^\theta_g(1-\delta) = \theta(k_g(1-\delta)k'_g))$. Thus $F(k^\theta_g, k'^\theta_g, a) = (f(k^\theta_g, a, A) - \theta f(k_g, a, A) - (1-\theta)f(\tilde{k}_g, a, A))p + \theta F(k_g, k'_g, a;) + (1-\theta)F(\tilde{k}_g, \tilde{k}'_g, a)$.

It is easy to check that integration preserves monotonicity and concavity under my assumptions. Now I need to prove that v is strictly concave in k_g by applying the Corollary 1 to the Contraction Mapping Theorem in SL. Let $C'(X_4) \subset C(X_4)$ be the set of bounded, continuous, weakly concave functions on X_4. Let $C''(X_4)$ bet the set of strictly concave functions. It is sufficient to show that $T[C'(X_4)] \subset C''(X_4)$ since $C'(X_4)$ is a closed subset of $C(X_4)$. Γ is convex-valued. Thus

$$k_0 \neq k_1 \qquad \theta \in (0,1) \qquad \text{and} \qquad k_\theta = \theta k_0 + (1-\theta)k_1.$$

Let $y_0 \in \Gamma$ and $y_1 \in \Gamma$, then $y_\theta = \theta y_0 + (1-\theta)y_1$, $y_\theta \in \Gamma$.

$$\begin{aligned} Tu(k_\theta, a, A, \mu) &\geq F(k_\theta, y_\theta, a, A, \mu)p + \beta \int u(y_\theta, a', A', \mu')Q(z, dz') \\ &> \theta[F(k_0, y_0)p + \beta \int u(y_0, a', A', \mu')Q(z, dz')] \\ &+ (1-\theta)[F(k_1, y_1)p + \beta \int u(y_1, a', A', \mu')Q(z, dz')] \\ &= \theta Tu(k_0, a, A, \mu) + (1-\theta)Tu(k_1, a, A, \mu) \end{aligned}$$

By applying Theorem 3(P. 117) in Berge(1963). The policy correspondence is a continuous and single valued function.

For strict increasing value function, I need to prove for every $(y, a; A, \mu)$, $F(\cdot, y, a; A, \mu)$ is strictly increasing. Pick $x_1, x_2 \in X$. Let $x_1 < x_2$. Want to prove $F(x_1, k_g', a;) < F(x_2, k_g', a;)$ for every $k_g' \in X$. If $rk_g' > x_2(1-\delta)$ or $rk_g' < x_1(1-\delta)$. it is obvious. When $rk_g' \in (x_1(1-\delta), x_2(1-\delta))$, $F(x_1, k_g', a;) = f(x_1, a, A)p + p_1(rk_g' - x_1(1-\delta)) + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p')$, $F(x_2, k_g', a;) = f(x_2, a, A)p + p_u(x_2(1-\delta) - rk_g') + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p')$. Since $p_1 < 0$. Done. When $k_g' = x_2(1-\delta)$, $F(x_2, k_g', a;) = f(x_2, a, A)p + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p')$,$F(x_1, k_g', a;) = f(x_1, a, A)p + p_1(k_g' - x_1(1-\delta)) + \beta(1-

$\delta)(1-\eta)p_c k_g' E(p_u' p')$. Thus $F(x_2, k_g', a) > F(x_1, k_g', a)$. When $k_g' = x_1(1-\delta)$, $F(x_2, k_g', a) = f(x_2, a, A)p + p_u(x_2(1-\delta) - k_g') + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p') > aA\tilde{x}_1 + \beta(1-\delta)(1-\eta)p_c k_g' E(p_u' p') = F(x_1, k_g', a)$. The argument is the same for strict concavity. Thus omitted. $\square$

Proof of Proposition 3.4.7. Pick $x = (a, A, \mu_1) \in Z \times \mathcal{D}_1$. The target levels and the cutoff values are defined as in main text. Let u_b^* and u_s^* be the maximums of above function. Define a function for inaction

$$\tilde{u}(x) = \beta\eta \int u(\frac{1}{r}k_g(1-\delta), x')Q(z, dz') + \beta(1-\eta)(1-\delta)^2 p_c k_g' E(p_u' p') \quad \text{(C.3)}$$

Equation C.3 describes the value when the plant does not adjust his capital except selling bad capital. Given $x = (k_g, a, A, \mu_1) \in E_g \times Z \times \mathcal{D}$, denote u_b the value of purchasing capital, $u_b = p_1(rk_1^* - k_g(1-\delta)) + u_b^*$. Denote u_s the value of selling good capital, $u_s = p_u(k_g(1-\delta) - rk_2^*) + u_s^*$. $x_1(x)$ is the cutoff values such that $u_b = \tilde{u}(x_1(x))$. $x_2(x)$ is the cutoff values such that $u_s = \tilde{u}(x_2(x))$. $x_1 = \frac{r}{1-\delta}k_1^*$. $x_2 = \frac{r}{1-\delta}k_2^*$.

Now I show that $k_1^* \leq k_2*$. I first show that u is proper. As u is monotonic, it is differentiable almost everywhere. Applying Corollary 25.1.1 in Rockafellar(1970), a concave function, which is differentiable at a given point, is proper. As u is continuous, it is closed. As u is bounded, the right and left derivative $u'_+(x)$ and $u'_-(x)$ are both bounded in the interior of E_g. By theorem 24.1, the left and right derivatives are non-increasing functions. $u'_+(x) \leq u'_-(x)$. Thus the multivalued mapping $\partial u(x) = \{x^* \in int(E_g) | u'_+(x) \leq x^* \leq u'_-(x)\}$. By Theorem 28.3, in the optimum, the subdifferential of the Lagrangian is zero.

$$rp_1 + \beta\eta \int \partial u(k_1^*)Q(z, dz') + \beta(1-\eta)(1-\delta)p_c E(p_u' p') = 0 \tag{C.4}$$

$$rp_u - \beta\eta \int \partial u(k_2^*)Q(z, dz') - \beta(1-\eta)(1-\delta)p_c E(p_u' p') = 0 \tag{C.5}$$

Suppose $k_1^* > k_2^*$. As $u'_+(x)$ and $u'_-(x)$ are both non-increasing functions, $u'_+(k_1*) \leq u'_-(k_1*) \leq u'_+(k_2^*) \leq u'_-(k_2^*)$. Thus $\partial u(k_1^*) \leq \partial u(k_2^*)$. After integration, $\int \partial u(k_1^*)Q(z, dz') \leq \int \partial u(k_2^*)Q(z, dz')$. But $-p_1 > p_u$. Contradiction. Suppose $k_1^* = k_2^*$, the plant can lose by buying and selling without changing the k_g'. Thus $u(k_g')$ is not well-defined function. Contradiction.

So $x_1 \leq x_2$. When $k_g < x_1$, the plant only buys capital or does not adjust. When $k_g > x_2$, the plant only sell capital or does not adjust. When plants buy or sell, the target levels do not depend on their predetermined capital stock. Since the policy function h_g is continuous on k_g as shown in Proposition 3.4.6, for $k_g < x_1$, the plant all invests or does not adjust. Otherwise, the policy function has jumps. Similar, when $k_g > x_2$, the plant all sells or does not adjust.

When $k_g < x_1$, plants decide to invest if the marginal benefit to invest $rp_1 + \beta\eta \int \partial u(k_g) + \beta(1-\eta)(1-\delta)p_c E(p_u' p')$ is positive. According to (C.4) and u'_+ is an non-increasing function, $rp_1 + \beta\eta \int \partial u'_+(k_g) + \beta(1-\eta)(1-\delta)p_c E(p_u' p') \geq 0$. That is , plants are indifferent or prefer to invest. If all plants with $k_g < x_1$ are indifferent, the value of investing equals to the value of inaction. Thus $p_1 = \frac{\beta\eta(Eu(k_1^*) - Eu(\frac{1}{r}k_g(1-\delta))) + \beta(1-\eta)(1-\delta)p_c(k_1^* - \frac{1}{r}k_g(1-\delta))E(p'p_u')}{rk_1^* - k_g(1-\delta)}$ for all $k_g < x_1$. Since u is strictly concave, this can't be true. Thus there must have some point with strict inequality. Thus at least the plant prefers to invest at one level of capital stock.

Therefore plants invest when $k_g < x_1$, as the policy function is continuous. The argument for selling decisions when $k_g > x_2$ is same. If $k_g \in (x_1, x_2)$, the plant can't buy or sell, thus not adjust.

□

C.3 A Characterization of the Frictionless Model

The individual plant's problem:

$$\begin{aligned} v(k, a; A, \mu_n) = max_{k',n} aAf(k, n) - wn + rk' - k(1-\delta) \\ + d(A, \mu_n)\eta \int v(k', a'; A', \mu_n')Q(z, dz') + d(A, \mu_n)(1-\eta)p_{cu}k' \end{aligned} \tag{C.6}$$

where $k \in [0, \bar{k}_n]$. p_{cu} is the discount of selling price for exiting firms. The representative household's problem is the same as the baseline model. The law of motions for distribution μ_n over (k, a) is $\mu_n' = H_n(A, \mu_n)$. The good market clearing condition is:

$$C(A, \mu_n) = \eta \int aAf(k, n)d\mu_n - \eta \int (rk' - k(1-\delta))d\mu_n + p_{cu} \int k(1-\delta)(1-\eta)d\mu_n \tag{C.7}$$

Definition 5. *The recursive competitive equilibrium for the economy*

$(\beta, f, Q, \eta, U, \delta, p_{cu}, \alpha, \nu)$

is a set of functions $(p, w, H_v, u, h_n, n, N^s, C, N)$ *such that: 1)* (u, h_n, n) *solve plants' problem given* (p, w)*; 2)* (C, N^s) *solve household's problem C.6 given* (w, p)*; 3) Good market clearing condition (C.7) satisfies; 4) Labor market clears; 5)*H_n *is generated from the equilibrium price, decision rules and exogenous aggregate shock.*

The law of motions H_n can be derived similarly from equation (3.16) by substituting h_n for h_g and changing the feasibility set for k.

C.4 A Characterization of the Fixed Irreversibility Model

The individual firm's problem:

$$\begin{aligned} & v(k, a; A, \mu_f) = max_{k',n} aAf(k, n) - wn + \mathbf{1}_{rk'>k(1-\delta)}(rk' - k(1-\delta)) \\ & + \mathbf{1}_{rk'\le k(1-\delta)} p_u(k(1-\delta) - rk') + d(A, \mu_f)\eta \int v(k', a'; A', \mu'_f)Q(z, dz') \\ & + d(A, \mu_f)(1-\eta)p_c k' E(p'_u p') \end{aligned} \tag{C.8}$$

where $k \in [0, \bar{k}_f]$. p_u is the degree of fixed irreversibility. The representative household's problem is the same as the baseline model. The law of motions for distribution μ_f over (k, a) is $\mu'_f = H_f(A, \mu_f)$. The good market clearing condition is:

$$\begin{aligned} C(A, \mu_f) = \eta \int aAf(k, n)d\mu_f + p_u \eta \int \mathbf{1}_{rk'\le k(1-\delta)}(k(1-\delta) - rk')d\mu_f \\ - \eta \int \mathbf{1}_{rk'>k(1-\delta)}(rk' - k(1-\delta))d\mu_f + p_c \int k(1-\delta)(1-\eta)d\mu_f \end{aligned} \tag{C.9}$$

Definition 6. *The recursive competitive equilibrium for the economy* $(\beta, f, Q, \eta, U, \delta, p_u^f, p_c, \alpha, \nu)$ *is a set of functions* $(p, w, H_f, u, h_f, n, N^s, C, N)$ *such that: 1)* (u, h_f, n) *solve plants' problem C.8 given* (p, w)*; 2)* (C, N^s) *solve household's problem given* (w, p)*; 3) Good market clearing condition (C.9) satisfies; 4) Labor market clears; 5)*H_f *is generated from the equilibrium price, decision rules and exogenous aggregate shock.*

The law of motions H_f can be derived similarly from equation (3.16) by substituting h_f for h_g and changing the feasibility set for k. In equilibrium, the firm's optimal investment decision also follows a two-sided (s, S) rules. The proof is similar to that in the baseline model.

C.5 Numerical Details

The individual plant's problem has $(k_g, a; K_g, A)$. a is a discrete variable. The rest are continuous variables. The state space is discretized as follows:

1. $n_k = 60$ is the number of the grid points for k_g between $[1.0^{-5}, 4]$, with log-equal-spaced grid width.
2. $n_{idio} = 6$ is the number of the grid points for idiosyncratic shocks with log-equal-spaced width. The total grid width is two times standard deviation. I compute the transition matrix using Tauchen (1986)'s method.
3. $n_a = 7$ is the number of the grid points for aggregate good capital stock between $[0.57, 1.178]$, equal-spaced.
4. $n_{agg} = 11$ is the number of the grid points for aggregate technology shock in $[0.92, 1.08]$, with more grids in the middle. I use seven integration nodes for the Gauss-Hermite Quadrature.

The plant's dynamic programming problem is computed with value function iteration and Howard Improvement. That is, after one period value function iteration, I use twenty periods policy improvement. The Newton method is employed to find the optimum in each grid points. I also compare the results with Golden section search. They are very similar. Since k_g, K_g and A are continuous, the in-between values are interpolated with a three-dimensional cubic spline, with a "not-a-knot" condition as stated in Kahn and Thomas(2003).

The simulation approach follows the method in Bachman, Caballero and Engel(2008). Beginning with an exogenous capital distribution and stationary distribution for idiosyncratic shock, the real distribution is computed every period according to the evolutions of idiosyncratic shock and aggregate shock. When the mass for some capital stock is less than 1.0^{-10}, it is taken out of the economy and I rescale the total mass accordingly.

In each simulation period, the equilibrium p and p_u are characterized as the root for two equations with two unknowns, the new good market clearing condition and consistent belief condition. These two equations are derived after the decision rules, previous distribution, x and λ. I use bisection method to find the root: In the upper loop, I need to find two boundary prices p_l and p_h which bracket the equilibrium p. Pick one p_l, in the lower loop, I need to find the equilibrium p_u which satisfies the consistent belief condition with p_l. After employing the bisection method for $p^*_{u,l}$, The aggregate variables under p_l are revealed. Compare the two prices to see whether the interval includes a zero-root. If does, apply the bisection method to find the root, otherwise change the boundary. The degree of precision for finding the root is 5.0^{-6}. The economy is simulated for 500 periods where the first 100 periods are dropped. The stopping criteria for the metric between two successive approximate laws of motions in 1.0^{-5}.

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Vita

Shaojin Li received the Bachelor of Engineering in Industrial Foreign Trade from the Beijing Institute of Technology, Beijing, China in July, 1999. She worked as a sales assistant for some small companies in Beijing before entering Peking University, Beijing, China. She received the degree of Master of Arts in Finance from China Center for Economic Research, Peking University in June, 2004. She entered the graduate school of the University of Texas at Austin in August, 2004.

Permanent address: 1-301 7th Building Yinghua Xiaqu
Heping Street, Chaoyang District, Beijing China

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